CS1800 Discrete Structures *Out: October 4, 2026*

Instructor Solutions to Problem Set 3

Strange & Varma *Due: October 11, 2026*

- This assignment is **due on October 11, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Two's complement) 30

Part A: Convert the decimal numbers -34 and -21 to two's complement, assuming an eight bit two's complement representation. Show your work, including how you get from the positive integers to their negative counterparts.

Solution:

$(34)_{10} = (00100010)_2$, so $-34 = \overline{00100010} + 1 = 11011101 + 1 = 11011110$

$(21)_{10} = (00010101)_2$, so $-21 = \overline{00010101} + 1 = 11101010 + 1 = 11101011$. ■

Part B: Convert the following numbers to decimal assuming that negative numbers are in two's complement form; 11011101, 00011011, 10001110

Solution:

11011101 in decimal:

$$\begin{aligned} 11011101 &\longrightarrow 00100010 && \text{Complement} \\ &\longrightarrow 00100011 && \text{Add 1} \\ &\longrightarrow -35 && \text{Decimal conversion} \end{aligned}$$

00011011 in decimal:

$$00011011 \longrightarrow 27 \qquad \text{Decimal Conversion (non-negative number)}$$

10001110 in decimal:

$$\begin{aligned} 10001110 &\longrightarrow 01110001 && \text{Complement} \\ &\longrightarrow 01110010 && \text{Add 1} \\ &\longrightarrow -114 && \text{Decimal conversion} \end{aligned}$$

■

Part C: We have the number 10110000 and we are not sure what convention is being used: i) what is the number assuming signed magnitude representation, and ii) what is the number if we assume two's complement?

Solution:

Signed Magnitude:

$$\begin{aligned} 10110000 &\longrightarrow -(00110000)_2 && \text{Sign bit} \\ &\longrightarrow -48 && \text{Decimal conversion} \end{aligned}$$

Two's Complement:

$$\begin{aligned} 10110000 &\longrightarrow 01001111 && \text{Complement} \\ &\longrightarrow 01010000 && \text{Add 1} \\ &\longrightarrow -80 && \text{Decimal Conversion} \end{aligned}$$

■

Part D: Sum the numbers you produced in **Part A** using binary addition, and verify that the result is equal to the binary for -55. Show your work.

Solution:

The binary representations we got in **Part A** were 11011110 and 11101011.

Carry	1	**1**	**1**	1	1	1	1		
		1	1	0	1	1	1	1	0
+		1	1	1	0	1	0	1	1
	1	1	1	0	0	1	0	0	1

Since the last two carry bits are the same, we can ignore the overflow bit, which gives us 11001001 as the two's complement representation of the sum. 11001001 flipped is 00110110; adding 1 gets us 00110111 = 1 + 2 + 4 + 16 + 32 = 55. So this is the binary for -55, as expected. ■

Part E: What is the sum, in decimal, of the sixteen-bit numbers encoded in two's complement 1111111111111111 and 0000000000001111? Show your work or explain your reasoning.

Solution:

Performing summation in binary:

Carry	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+		0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0

Since the last two carry bits are the same, we can ignore the overflow bit, which gives us 0000000000001110 as the two's complement representation of the sum. This is equal to $(1110)_2 = 8 + 4 + 2 = (14)_{10}$.

Shortcut approach: The all one's number must be equal to -1, since adding one to it would produce 0. The other number is 8+4+2+1=15. So their sum, in decimal, must be 14.

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Problem 2 (Converting to Logic) 15+(4+6)+5

1. Show or describe in English the elements that make up the following set:

Solution:

- $A = \{x \in \mathbb{Z} \mid x^2 < 6x\}$

  $A = \{1, 2, 3, 4, 5\}$.
  All integers such that the square of the integer is smaller than six times the integer.

- $B = \{x \in S \mid 3x \in S\}$ where $S = \{2, 3, 5, 9\}$

  $B = \{3\}$.
  The set of elements from S such that both the element and three times the element belong in S.

- $C = \{(x, y) \mid (x + y)^2 > 100 \text{ and } x, y \in \mathbb{N}\}$

  All pairs of natural numbers (x, y) such that the sum of x and y is more than 10.[1]

- $D = \{n^2 \in \mathbb{R} \mid n < |n|\}$

  The set of squares of real numbers such that the real number is strictly smaller than the absolute value of the real number.[2]

- Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universe. Let $A = \{1, 4, 6\}$ and $B = \{1, 2, 3, 7, 9\}$ be two subsets of U. What are the elements of the set $\overline{\overline{A} \cap \overline{B}}$?
  $\overline{\overline{A} \cap \overline{B}} = \overline{\overline{A \cup B}} = A \cup B = \{1, 2, 3, 4, 6, 7, 9\}$?

■

2. Write the following description of collections using set notation:

Solution:

- All positive real numbers in which the absolute value of the square root of the number is less than or equal to 10.

$$\{x \in \mathbb{R}^+ \ : \ |\sqrt{x}| \leq 10\}$$

- All pairs of real numbers where their sum is 12.

$$\{(x, y) \mid x \in \mathbb{R}, y \in \mathbb{R}, (x + y) = 12\}$$

■

3. For each of the following statements, state whether it is TRUE or FALSE. Show your work and explain your answer.

[1] All lattice points in the plane which are more than 100 away from the origin.
[2] This is the same as all positive real numbers, $\mathbb{R}^+$.

Solution:

- $-22 \in \{4x - 10 \ : \ x \in \mathbb{Z}\}$

  True. Solving $-22 = 4x - 10$ for x gives $x = -3$ and $-3 \in \mathbb{Z}$.

- $-22 \in \{4x - 10 \ : \ x \in \mathbb{N}\}$

  False. Solving $-22 = 4x - 10$ for x gives $x = -3$ and $-3 \notin \mathbb{N}$.

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Problem 3 (Venn Diagrams) 24

Shade the regions for the following sets in a Venn diagram.

Solution:

1. $(A \cap B) \cup C$

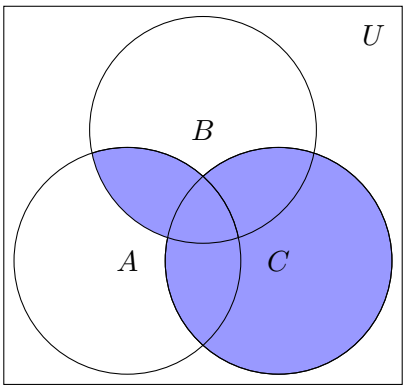


Figure 1: $(A \cap B) \cup C$

2. $(B \setminus A) \cap C$

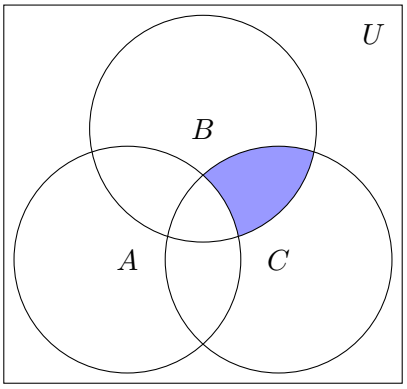


Figure 2: $(B \setminus A) \cap C$

3. $(A \cup B) \cap \overline{C}$

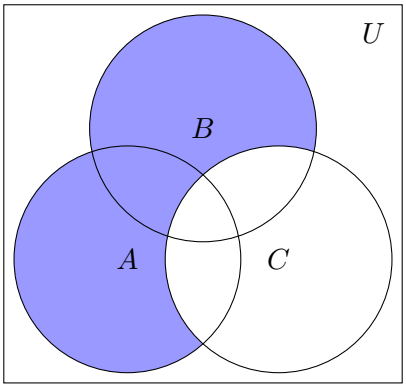


Figure 3: $(A \cup B) \cap \overline{C}$

4. $(A \cup C)\Delta B$

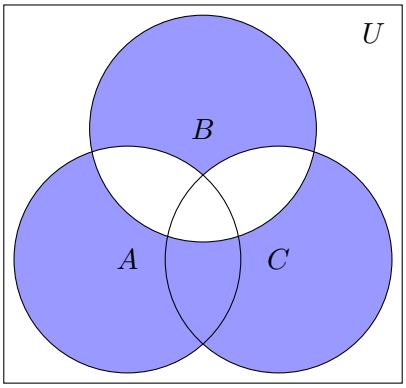


Figure 4: $(A \cup C)\Delta B$

5. $(A\Delta B\Delta C) \cup \overline{A}$

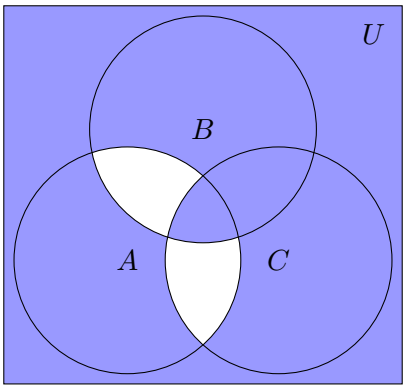


Figure 5: $(A\Delta B\Delta C) \cup \overline{A}$

6. $(A \cup \overline{A}) \setminus (A \cap B \cap C)$

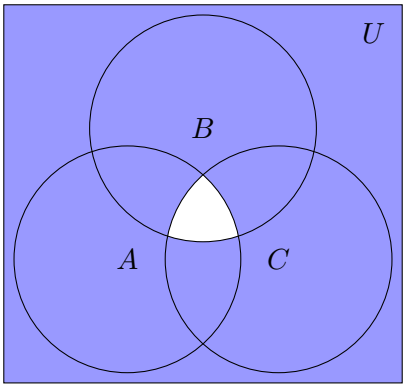


Figure 6: $(A \cup \overline{A}) \setminus (A \cap B \cap C)$

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Problem 4 (Simplifying Sets) 2+4+6+8

Simplify each of the following expressions where U is the universal set which contains all elements. Your final simplification should only use unions, intersections and complements and in particular it should not use the Δ operator.

Solution:

1. $A \cap A = A$ [Idempotence]
2. $\overline{\overline{A} \cap \overline{B}} \cap U = \overline{\overline{A} \cap \overline{B}} = \overline{\overline{A \cup B}} = A \cup B$ [Identity, De Morgan, Double Negation]
3. $(A \cup A) \cap (B \cup \overline{A}) = A \cap (B \cup \overline{A}) = (A \cap B) \cup (A \cap \overline{A}) = (A \cap B) \cup \emptyset = (A \cap B)$
   [Idempotence, Distributive, Complement, Identity]
4. $$\begin{aligned}
   & \overline{\overline{A \cap \overline{A} \cap A} \cap B \cap \overline{\overline{C} \cap \overline{A}}} \\
   &= \overline{\overline{\emptyset \cap A} \cap B \cap \overline{\overline{C} \cap \overline{A}}} && \text{Complement} \\
   &= \overline{\overline{\emptyset \cap A} \cap B \cap \overline{\overline{C} \cup \overline{A}}} && \text{DeMorgan's} \\
   &= \overline{\overline{\emptyset \cap A} \cap B \cap (C \cup A)} && \text{Double Negation} \\
   &= \overline{\overline{\emptyset} \cap B \cap (C \cup A)} && \text{Domination} \\
   &= \overline{U \cap B \cap (C \cup A)} && \text{Definition: } \overline{\emptyset} = U \\
   &= \overline{B \cap (C \cup A)} && \text{Identity}
   \end{aligned}$$

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