CS1800 Discrete Structures *Out: September 19, 2026*

Problem Set 2

Strange & Varma *Due: September 25, 2026*

- This assignment is **due on September 25, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Logical Equivalence) 20 pts

Are the following statements logically equivalent, if so, show their truth table, if not, show on what inputs they do not agree.

Part A: $A \wedge B$, $(A \vee B) \wedge B$

Part B: $\neg(A \wedge B) \vee C$, $(\neg A \vee \neg B) \vee C$

Part C: $\neg A \vee T$, A

Part D: $(A \wedge B) \wedge (B \wedge \neg C)$, $\neg((\neg A \vee \neg B) \vee (\neg B \vee C))$

Part E: $(\neg B \vee \neg(C \wedge D))$, $\neg(C \vee B) \wedge \neg(B \vee D)$

Problem 2 (Equivalencia) 40 pts

Prove the following boolean expressions/formulae are equivalent, indicating which law is used at each step:

Part A: $(B \wedge C) \vee (B \wedge \neg C) \vee (\neg B \wedge A) \equiv (A \vee B)$

Part B: $(a \wedge \neg b) \vee (a \wedge \neg c) \equiv a \wedge \neg(b \wedge c)$

Part C: $\neg(p \wedge \neg(q \vee \neg(p \wedge q))) \equiv T$

Part D: $(p \wedge q) \vee \neg(p \vee \neg q) \vee (q \wedge r) \equiv q$

Problem 3 (Multiple Choice Logic) 20 pts

Which answer is correct?

a. All of the below

b. None of the below

c. All of the above

d. One of the above

e. None of the above

f. None of the above

Create a proposition for each of the option, and rewrite each statement in terms of the proposition using logical operators. As an example, let the first option be the proposition a, based on the statement it would equal $b \wedge c \wedge d \wedge e \wedge f$. Once you have all propositions listed in terms of the others, determine which option(s)/proposition(s) must be true and reason why that must be the case.

Problem 4 (Converting to Logic) **25 pts**

Consider the following, which are potentially Akshar's interests:

- H = "High Fantasy genre."
- S = "Science Fiction genre."
- K = "K-pop."
- R = "Rubik's cubes."
- B = "Binging shows."

Write a logical expression (using $\wedge, \vee, \neg$ operations) for the requirements under the following conditions:

1. Akshar loves the Science Fiction genre but does not listen to K-pop.
2. Akshar is interested in binging shows and he likes at least one of Science Fiction and High Fantasy genres.
3. Akshar either listens to K-pop or binges shows but not both.
4. Akshar likes at least two of the following: High Fantasy, Science Fiction, Rubik's cubes.
5. Exactly 2 of the following statements about Akshar are true: solves Rubik's cubes, loves Science Fiction, binges shows.

Problem 5 (Normal Form) — 20 pts

Part A: Are the following expression in CNF? If not, identify why.

(i) $\neg A \wedge \neg B \wedge B \vee C$

(ii) $(\neg A \vee (B \vee C)) \wedge D$

(iii) $(A \wedge B) \vee (B \wedge C) \vee \neg D$

Part B: Are the following expressions in DNF? If not, identify why.

(i) $\neg(\neg B) \wedge (B \vee D)$

(ii) $(B \wedge \neg D \wedge C \wedge E \wedge B \wedge F) \vee A$

(iii) $(B \wedge D) \vee (C \wedge D) \vee \neg(B)$

Part C: Write a DNF formula that is true when exactly three of the four inputs A, B, C, D are false.

Problem 6 (Express using logic notation) 30

In Figure 1, we have a Tarski world; the domains for all variables in this problem consists of the objects (individual shapes) in the Tarski world. Every object has a shading, it is either fully shaded, half shaded, or clear (no shading).

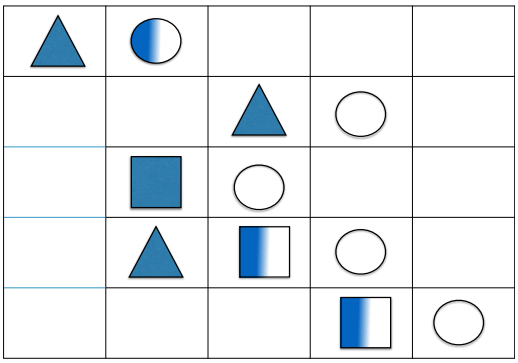


Figure 1: Figure of A Tarski World, a tool for understanding logical statements invented by Jon Barwise and John Etchemendy, named after Alfred Tarski

For each statement below, write the contrapositive, the converse, and the inverse using english (no logical operators needed). Tell which of the four statements (the three you wrote, in addition to the original statement) is true or false. If a statement is false, provide a counter-example.

Example:

- Original Conditional Statement ($P \implies Q$):
  - If an object is clear, then it is a circle.
  - True.
- Contrapositive ($\neg Q \implies \neg P$):
  - If an object is not a circle then it is not clear.
  - True.
- Converse ($Q \implies P$)
  - If an object is a circle, then it is clear.
  - False, circle in the first row is shaded.
- Inverse ($\neg P \implies \neg Q$)
  - If an object is not clear then it is not a circle.
  - False, circle in the first row is shaded.

Use the following predicates when describing an object: Circle, Square, Triangle, Fully Shaded, Half Shaded, Clear.

1. If an object is fully shaded, then it is a triangle.
2. If an object is a rectangle, then it is fully shaded or half shaded.
3. If an object is clear or half shaded, then it is a circle.