CS1800 Discrete Structures *Out: September 19, 2026*

Instructor Solutions to Problem Set 2

Strange & Varma *Due: September 25, 2026*

- This assignment is **due on September 25, 2026 at 11:59 PM** via Gradescope. You may submit up to 48 hours late with no penalty.
- Your solutions must be handwritten, with your name on the first page. All your work must be shown even if the problem doesn't specifically ask for it.
- Discussing problems with classmates is allowed but all solutions must be written by yourself, in your own words.
- Copying solutions from a classmate, an online source, or AI tools, is a violation of our academic integrity policy. Do *not* submit anything you cannot explain. Please list all your collaborators and online reference sources in your solution for each problem.
- If you get stuck on a homework problem, come by office hours or post on Campuswire! We recommend you spend about 30 minutes trying to figure out a problem, and then ask for help. We'll be happy to clarify material from class and algorithm concepts, but we will not give out solutions or confirm your answers are correct.

Problem 1 (Logical Equivalence) 20 pts

Are the following statements logically equivalent, if so, show their truth table, if not, show on what inputs they do not agree.

Part A: $A \wedge B$, $(A \vee B) \wedge B$

Part B: $\neg(A \wedge B) \vee C$, $(\neg A \vee \neg B) \vee C$

Part C: $\neg A \vee T$, A

Part D: $(A \wedge B) \wedge (B \wedge \neg C)$, $\neg((\neg A \vee \neg B) \vee (\neg B \vee C))$

Part E: $(\neg B \vee \neg(C \wedge D))$, $\neg(C \vee B) \wedge \neg(B \vee D)$

Solution:

Part A: No, If A is false and B is true, then $A \wedge B$ is false but $(A \vee B) \wedge B$ is true.

Part B: Yes. $\neg(A \wedge B) \vee C$, $(\neg A \vee \neg B) \vee C$

A	B	C	$\neg(A \wedge B) \vee C$	$(\neg A \vee \neg B) \vee C$
F	F	F	T	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

Part C: No. If A is false, $\neg A \vee T$ is true but A is false.

Part D: Yes. $(A \wedge B) \wedge (B \wedge \neg C)$, $\neg((\neg A \vee \neg B) \vee (\neg B \vee C))$

A	B	C	$A \wedge B) \wedge (B \wedge \neg C)$	$\neg((\neg A \vee \neg B) \vee (\neg B \vee C))$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	F	F
T	T	F	T	T
T	T	T	F	F

Part E: No. If B and D are true but C is false, then $(\neg B \vee \neg(C \wedge D))$ evaluates to true but $\neg(C \vee B) \wedge \neg(B \vee D)$ evaluates to false.

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Problem 2 (Equivalencia) 40 pts

Prove the following boolean expressions/formulae are equivalent, indicating which law is used at each step:

Part A: $(B \wedge C) \vee (B \wedge \neg C) \vee (\neg B \wedge A) \equiv (A \vee B)$

Solution:

The annotation on each line describes how we reached that line from the line above. The orange/red colored text is where the law was applied on the line above to reach the line below.

$$
\begin{aligned}
& (B \wedge C) \vee (B \wedge \neg C) \vee (\neg B \wedge A) && \text{Start point} \\
\equiv\ & \Big(B \wedge (C \vee \neg C)\Big) \vee (\neg B \wedge A) && \text{Distributive law} \\
\equiv\ & (B \wedge T) \vee (\neg B \wedge A) && \text{Negation/Complement law} \\
\equiv\ & B \vee (\neg B \wedge A) && \text{Identity law} \\
\equiv\ & (B \vee \neg B) \wedge (B \vee A) && \text{Distributive law} \\
\equiv\ & T \wedge (B \vee A) && \text{Negation/Complement law} \\
\equiv\ & (B \vee A) && \text{Identity law} \\
\equiv\ & (A \vee B) && \text{Commutative law; end point}
\end{aligned}
$$

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Part B: $(a \wedge \neg b) \vee (a \wedge \neg c) \equiv a \wedge \neg(b \wedge c)$

Solution:

$$
\begin{aligned}
& (a \wedge \neg b) \vee (a \wedge \neg c) && \text{Start} \\
\equiv\ & a \wedge (\neg b \vee \neg c) && \text{Distributive Law} \\
\equiv\ & a \wedge \neg(\neg\neg b \wedge \neg\neg c) && \text{De Morgan's Law} \\
\equiv\ & a \wedge \neg(b \wedge c) && \text{Double Negation Law}
\end{aligned}
$$

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Part C: $\neg(p \wedge \neg(q \vee \neg(p \wedge q))) \equiv T$

Solution:

$$
\begin{aligned}
& \neg(p \wedge \neg(q \vee \neg(p \wedge q))) && \text{Start} \\
\equiv\ & \neg(p \wedge \neg(q \vee \neg p \vee \neg q)) && \text{De Morgan's Law} \\
\equiv\ & \neg(p \wedge \neg(T \vee \neg p)) && \text{Complement Law} \\
\equiv\ & \neg(p \wedge \neg T) && \text{Annihilator Law} \\
\equiv\ & \neg(p \wedge F) \\
\equiv\ & \neg F && \text{Annihilator Law} \\
\equiv\ & T
\end{aligned}
$$

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Part D: $(p \wedge q) \vee \neg(p \vee \neg q) \vee (q \wedge r) \equiv q$

Solution:

$$
\begin{array}{rll}
 & (p \wedge q) \vee \neg(p \vee \neg q) \vee (q \wedge r) & \text{Start} \\
\equiv & (p \wedge q) \vee (\neg p \wedge \neg\neg q) \vee (q \wedge r) & \text{De Morgan's Law} \\
\equiv & (p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r) & \text{Double Negation Law} \\
\equiv & (q \wedge (p \vee \neg p)) \vee (q \wedge r) & \text{Distributive Law} \\
\equiv & (q \wedge T) \vee (q \wedge r) & \text{Complement Law} \\
\equiv & q \vee (q \wedge r) & \text{Identity Law} \\
\equiv & q & \text{Absorption law}
\end{array}
$$

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Problem 3 (Multiple Choice Logic) 20 pts

Which answer is correct?

a. All of the below

b. None of the below

c. All of the above

d. One of the above

e. None of the above

f. None of the above

Create a proposition for each of the option, and rewrite each statement in terms of the proposition using logical operators. As an example, let the first option be the proposition a, based on the statement it would equal $b \wedge c \wedge d \wedge e \wedge f$. Once you have all propositions listed in terms of the others, determine which option(s)/proposition(s) must be true and reason why that must be the case.

Solution:

The first "None of the above", option e. The following are "English" reasoning, without using formal logical propositions, operators or reasoning.

a. "All of the below": Not true. If it were true, then b. "None of the below" would be true, which would force the rest to be false. So options c, d, e, f have to be both true and false which isn't possible. So option a must be false instead.

b. "None of the below": Not true. If true, then option d. "One of the above" would be false, which requires option b. to be false leading to a contradiction.

c. "All of the above": Isn't true. We just found that both of them are false. Also, the two options above contradict each other, so both can't be true.

d. "One of the above": Not true, since we argued why all of the above options are false.

e. "None of the above": True, since we already argued why each of them is False.

f. "None of the above": False, since the one above is true.

Formally, the propositions can be expressed as:

- $a = b \wedge c \wedge d \wedge e \wedge f$
- $b = (\neg c \wedge \neg d \wedge \neg e \wedge \neg f)$
- $c = a \wedge b$
- $d = a \vee b \vee c$
- $e = (\neg a \wedge \neg b \wedge \neg c \wedge \neg d)$
- $f = (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

Using the above formalized logical expressions, we can determine the truthfulness of each of the variables as follows.

- a cannot be true since:

$$\begin{aligned} a &= b \wedge c \wedge d \wedge e \wedge f \\ &= (\neg c \wedge \neg d \wedge \neg e \wedge \neg f) \wedge c \wedge d \wedge e \wedge f \\ &= (c \wedge \neg c) \wedge (d \wedge \neg d) \wedge (e \wedge \neg e) \wedge (f \wedge \neg f) && \text{Associative law} \end{aligned}$$

  In the final expression for a, each of the clauses is always false making the overall statement false.

- b cannot be true since $b = T \implies d = F$, however $d = F \implies b = F$ which by chaining gives us $b = T \implies b = F$ which is absurd.

- c cannot be true since we already concluded that both of a and b are false and thus $c = a \wedge b$ must be false.

- d cannot be true since we already concluded that all of a, b, c are false and thus $d = a \vee b \vee c$ must also be false.

- e is true since we concluded that all of a, b, c, d are false which makes $e = (\neg a \wedge \neg b \wedge \neg c \wedge \neg d) = T \wedge T \wedge T \wedge T = T$.

- f is false since we've concluded that all of a, b, c, d are false and $e = T$ which makes $f = (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e) = T \wedge T \wedge T \wedge T \wedge F = F$.

This gives us the same result as our informal/intuitive reasoning but the formal approach is less error prone. ■

Problem 4 (Converting to Logic) **25 pts**

Consider the following, which are potentially Akshar's interests:

- H = "High Fantasy genre."
- S = "Science Fiction genre."
- K = "K-pop."
- R = "Rubik's cubes."
- B = "Binging shows."

Write a logical expression (using $\wedge, \vee, \neg$ operations) for the requirements under the following conditions:

1. Akshar loves the Science Fiction genre but does not listen to K-pop.
2. Akshar is interested in binging shows and he likes at least one of Science Fiction and High Fantasy genres.
3. Akshar either listens to K-pop or binges shows but not both.
4. Akshar likes at least two of the following: High Fantasy, Science Fiction, Rubik's cubes.
5. Exactly 2 of the following statements about Akshar are true: solves Rubik's cubes, loves Science Fiction, binges shows.

Solution:

1. $S \wedge \neg K$
2. $B \wedge (S \vee H)$
3. $(K \vee B) \wedge \neg(K \wedge B)$
4. $(H \wedge S) \vee (S \wedge R) \vee (R \wedge H)$
5. $(R \wedge S \wedge \neg B) \vee (R \wedge \neg S \wedge B) \vee (\neg R \wedge S \wedge B)$

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Problem 5 (Normal Form) 20 pts

Part A: Are the following expression in CNF? If not, identify why.

(i) $\neg A \wedge \neg B \wedge B \vee C$

(ii) $(\neg A \vee (B \vee C)) \wedge D$

(iii) $(A \wedge B) \vee (B \wedge C) \vee \neg D$

Solution:

Part A: No– there is a disjunction between clauses: $B \vee C$

Part B: No– not all terms are in a clean single clause but as a clause-in-a-clause situation: $(\neg A \vee (B \vee C))$
Yes– if we just remove the inner parenthesis, then $(\neg A \vee B \vee C)$ is a valid clause and together with the rest forms CNF.

Part C: No– it is in fact a DNF.

■

Part B: Are the following expressions in DNF? If not, identify why.

(i) $\neg(\neg B) \wedge (B \vee D)$

(ii) $(B \wedge \neg D \wedge C \wedge E \wedge B \wedge F) \vee A$

(iii) $(B \wedge D) \vee (C \wedge D) \vee \neg(B)$

Solution:

(i) No– a NOT is outside a clause: $\neg(\neg B)$

(ii) Yes.

(iii) No– the not is outside a clause: $\neg(B)$
Yes– if this is read the same as: $\neg(B) \longrightarrow (\neg B)$ then it is a valid single term clause.

■

Part C: Write a DNF formula that is true when exactly three of the four inputs A, B, C, D are false.

Solution:

$$(A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge B \wedge \neg C \wedge \neg D)$$
$$\vee(\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge \neg C \wedge D)$$

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Problem 6 (Express using logic notation) 30

In Figure 1, we have a Tarski world; the domains for all variables in this problem consists of the objects (individual shapes) in the Tarski world. Every object has a shading, it is either fully shaded, half shaded, or clear (no shading).

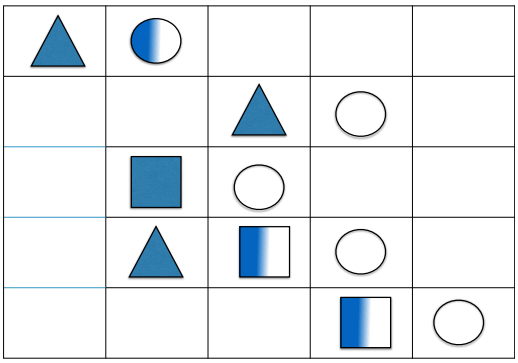


Figure 1: Figure of A Tarski World, a tool for understanding logical statements invented by Jon Barwise and John Etchemendy, named after Alfred Tarski

For each statement below, write the contrapositive, the converse, and the inverse using english (no logical operators needed). Tell which of the four statements (the three you wrote, in addition to the original statement) is true or false. If a statement is false, provide a counter-example.

Example:

- Original Conditional Statement ($P \implies Q$):
  - If an object is clear, then it is a circle.
  - True.
- Contrapositive ($\neg Q \implies \neg P$):
  - If an object is not a circle then it is not clear.
  - True.
- Converse ($Q \implies P$)
  - If an object is a circle, then it is clear.
  - False, circle in the first row is shaded.
- Inverse ($\neg P \implies \neg Q$)
  - If an object is not clear then it is not a circle.
  - False, circle in the first row is shaded.

Use the following predicates when describing an object: Circle, Square, Triangle, Fully Shaded, Half Shaded, Clear.

1.

Solution:

Original: If an object is fully shaded, then it is a triangle.
False: Square in row 3 is fully shaded but is not a triangle.

Contrapositive: If an object is not a triangle, then it is not fully shaded.
False: Square in row 3 is fully shaded but is not a triangle.

Converse: If an object is a triangle, then it is fully shaded.
True. All triangles are in fact fully shaded.

Inverse: If an object is not fully shaded, then it is not a triangle.
True. None of the objects that are clear or half shaded are triangles.

■

2.

Solution:

Original: If an object is a rectangle, then it is fully shaded or half shaded.
True: All rectangle are fully shaded or half shaded.

Contrapositive: If an object is not fully shaded nor half shaded, then it is not a rectangle.
True: None of the clear objects are rectangles.

Converse: If an object is fully shaded or half shaded, then it is a rectangle.
False. All the triangles are fully shaded but are not rectangles.

Inverse: If an object is not a rectangle, then it is not fully shaded nor half shaded.
False. No triangles is a rectangle but all triangles are fully shaded.

■

3.

Solution:

Original: If an object is clear or half shaded, then it is a circle.
False: Squares in the last two rows are half shaded but they are not circles.

Contrapositive: If an object is not a circle, then it is not clear nor half shaded.
False: The rectangles in the last two rows are not circles, but they are half shaded.

Converse: If an object is a circle, then it is clear or half shaded.
True. Every circle is in fact clear, except for the one in row 1 which is half shaded.

Inverse: If an object is not clear or half shaded, then it is not a circle.
True. All fully shaded objects are triangles or rectangles.

■