CS1800 Discrete Structures

Solutions to Exam Practice 2

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- **Actual vs. Practice:** The primary difference in the actual exam vs. this practice exam is the length. Each (sub)topic/concept will still show up in the actual exam but the number of times it shows up will be once instead of multiple occurrences as in this practice. The relative proportion of points will be similar in the actual exam, but the total will be different. The instructions below will also be the same in the actual exam.

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID in the blank space provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **11** problems totaling **290** points. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. Show your work, as partial credit may be given. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring one 8.5x11–inch cheat sheet, with anything written/typed on it, only one side of the sheet can be used. No other materials or devices are permitted. No collaboration of any kind is permitted.
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Points	15	15	40	41	20	24	20	20	40	35	20	290

Best of luck!

Table 1: Laws of Set Operations. A, B, C are arbitrary sets; U is the universal set; $\emptyset$ is the empty set.

Commutative Laws	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative Laws	$(A \cap B) \cap C \equiv A \cap (B \cap C)$ $(A \cup B) \cup C \equiv A \cup (B \cup C)$
Distributive Laws	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Identity Laws	$A \cap U \equiv A$ $A \cup \emptyset \equiv A$
Annihiltor/Domination Laws	$A \cap \emptyset \equiv \emptyset$ $A \cup U \equiv U$
Idempotence Laws	$A \cap A \equiv A$ $A \cup A \equiv A$
Complement/Negation Laws	$A \cap A^C \equiv \emptyset$ $A \cup A^C \equiv U$
Double Negation Law	$\left(A^C\right)^C \equiv A$
Absorption Laws	$A \cap (A \cup B) \equiv A$ $A \cup (A \cap B) \equiv A$
De Morgan's Laws	$\overline{(A \cap B)} \equiv \overline{A} \cup \overline{B}$ $(A \cup B)^C \equiv A^C \cap B^C$

Problem 1 (6-bit Two's Complement) 5+5+5

In this problem, all two's complement is in 6-bit two's complement notation.

(a) Convert -30 into 6-bit two's complement notation.

Solution:

$(30)_{10} = (011110)_2$, so $-30 = \overline{011110} + 1 = 100001 + 1 = 100010$. ■

(b) The numbers $x = 111101$ and $y = 111001$ are in two's complement notation. Add these, verify whether there is overflow or not.

Solution:

Carry	1	1	1	0	0	1	
		1	1	1	1	0	1
+		1	1	1	0	0	1
	1	1	1	0	1	1	0

So the result is $x + y = 110110$. The two numbers were negative as is the result, so there is no overflow.

■

(c) Convert the number $z = 110101$ from two's complement into decimal.

Solution:

Converting to decimal is done via the same process; complement, then add 1:
$110101 \longrightarrow 001010 \longrightarrow -(001011)_2 \longrightarrow -(11)_{10}$

Alternatively, you can subtract 1 and then complement:
$110101 \longrightarrow 110100 \longrightarrow -(001011)_2 \longrightarrow -(11)_{10}$

Both give -11 as the decimal value. ■

Problem 2 (5-bit Two's Complement) 5+5+5

In this problem, all two's complement is in 5-bit two's complement notation.

(a) Convert -12 into 5-bit two's complement notation.

Solution:

$(12)_{10} = -(01100)_2$, so $-12 = \overline{01100} + 1 = 10011 + 1 = 10100$. ■

(b) The numbers $x = 11101$ and $y = 11110$ are in two's complement notation. Add these and verify whether there is overflow or not.

Solution:

Carry	1	1	1	0	0	
		1	1	1	0	1
+		1	1	1	1	0
	1	1	1	0	1	1

So the result is $x + y = 11011$. The two numbers were negative as is the result, so there is no overflow.

■

(c) Convert the number $z = 11010$ from two's complement into decimal.

Solution:

Converting to decimal is done via the same process; complement, then add 1:
$11010 \longrightarrow 00101 \longrightarrow -(00110)_2 \longrightarrow -(6)_{10}$

Alternatively, you can subtract 1 and then complement:
$11010 \longrightarrow 11001 \longrightarrow -(00110)_2 \longrightarrow -(6)_{10}$

Both give -6 as the decimal value. ■

Problem 3 (Laws of Set Operations) 4 x 10

Simplify the following expressions using laws of set operations while labeling the law used in each step. You do not need to specify when the commutative or associative laws are being used.

Note: In the actual exam, the format will be similar to the boolean laws question where you are asked to fill in gaps. The practice is formatted this way to give a slightly more comprehensive practice.

(a) $\overline{(\overline{B} \cup (B \cap \overline{C})) \cap \overline{C} \cap B \cap B \cap \overline{\overline{A}}}$

Solution:

$\overline{(\overline{B} \cup (B \cap \overline{C})) \cap \overline{C} \cap B \cap B \cap \overline{\overline{A}}}$	
$= \overline{\overline{C} \cap ((B \cap \overline{B}) \cup (B \cap B \cap \overline{C})) \cap B \cap \overline{\overline{A}}}$	Distributive
$= \overline{\overline{C} \cap B \cap (\overline{B} \cup \overline{C}) \cap B \cap \overline{\overline{A}}}$	Distributive
$= \overline{\overline{C} \cap B \cap B \cap \overline{\overline{A}}}$	Absorption
$= \overline{\overline{C \cup \overline{A}} \cap B \cap B}$	DeMorgan's
$= \overline{\overline{C \cup \overline{A}} \cap B}$	Idempotent
$= \overline{\overline{C \cup \overline{A}}} \cup \overline{B}$	DeMorgan's
$= C \cup A \cup \overline{B}$	Double Negation

■

(b) $(\overline{B} \cup (C \cap \overline{C} \cap B)) \cap (C \cup C \cup A)$

Solution:

$(\overline{B} \cup (C \cap \overline{C} \cap B)) \cap (C \cup C \cup A)$	
$= (\overline{B} \cup (\emptyset \cap B)) \cap (C \cup C \cup A)$	Complement
$= (\overline{B} \cup \emptyset) \cap (C \cup C \cup A)$	Domination
$= \overline{B} \cap (C \cup C \cup A)$	Identity
$= \overline{B} \cap (C \cup A)$	Idempotent

■

(c) $C \cup (((\overline{B} \cap A) \cup \overline{C} \cup \overline{A}) \cap (\overline{B} \cup A \cup A \cup \overline{B}))$

Solution:

$$
\begin{aligned}
& C \cup (((\overline{B} \cap A) \cup \overline{C} \cup \overline{A}) \cap (\overline{B} \cup A \cup A \cup \overline{B})) && \\
= {} & C \cup (((\overline{B} \cap A) \cup \overline{C} \cup \overline{A}) \cap (\overline{B} \cup A \cup A)) && \text{Idempotent} \\
= {} & (C \cup (\overline{B} \cap A) \cup \overline{C} \cup \overline{A}) \cap (C \cup \overline{B} \cup A \cup A) && \text{Distributive} \\
= {} & (U \cup (\overline{B} \cap A) \cup \overline{A}) \cap (C \cup \overline{B} \cup A \cup A) && \text{Complement} \\
= {} & U \cap (C \cup \overline{B} \cup A \cup A) && \text{Domination} \\
= {} & C \cup \overline{B} \cup A \cup A && \text{Identity} \\
= {} & C \cup \overline{B} \cup A && \text{Idempotent}
\end{aligned}
$$

■

(d) $\Big((B \cap A \cap C) \cup (C \cap B \cap \overline{A})\Big)^C$

Solution:

$$
\begin{aligned}
& \Big((B \cap A \cap C) \cup (C \cap B \cap \overline{A})\Big)^C && \\
= {} & \Big(B \cap ((A \cap C) \cup (C \cap \overline{A}))\Big)^C && \text{Distributive} \\
= {} & \Big(B \cap C \cap (A \cup \overline{A})\Big)^C && \text{Distributive} \\
= {} & \Big(B \cap C \cap U\Big)^C && \text{Complement} \\
= {} & \Big(B \cap C\Big)^C && \text{Identity}
\end{aligned}
$$

■

Problem 4 (Sets) 2+4+2+2+3+5+3+4+5+4+3+4

In this problem, the universe is the 10 digits $U = \{0, 1, 2, \ldots, 9\}$. We also have the sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 7\}$, $C = \{0, 2, 4, 8\}$ and $D = \{0, 1, 4, 9\}$.

(a) True/False: $2 \in B \cap C$.

Solution:

True. $B \cap C = \{2\}$, so 2 is definitely an element. ■

(b) Write the set $\{x \in D \mid x > 0\} \Delta \{k^2 \mid k \in B \cap C\}$ in Roster notation.

Solution:

Positive elements in D are $\{1, 4, 9\}$. Elements of $\{k^2 \mid k \in B \cap C\}$ are $\{4\}$ since 2 is the only element in $B \cap C$. Symmetric different of two sets contains elements found in exactly one of the sets. So in roster notation these are: $\{1, 9\}$ ■

(c) True/False: $(A \cup D) \subseteq U$.

Solution:

True. Since all elements of A and all elements of D are in the universe U, $A \cup D$ is a subset of U. ■

(d) True/False: $(B \cup \overline{A}) \in \mathcal{P}(U)$.

Solution:

True. All elements of B and of A^C are all in the universe U. So their union is definitely a subset of U and hence an element of the power set of the universe U. ■

(e) How many elements will be there in $A \times D$?

Solution:

$|A| = 4$ and $|D| = 4$ so $|A \times D| = 4 \times 4 = 16$. ■

(f) Write the set $A \times D \cap \{(n, n^2) \mid n \in U\}$ in set builder notation in as simplified a form as possible.

Solution:

$A \times D$ contains pairs where some number from 1 to 4 is paired with squares upto 9. The second set contains all pairs (n, n^2). The intersection will be (k, k^2) pairs with $k \in \{1, 2, 3\}$. In set builder notation, this is:

$$\left\{(k, k^2) \;\middle|\; k \in \{1, 2, 3\}\right\}$$

■

(g) What is the set: $U \setminus (B \cup D)$ in roster notation?

Solution:

Elements not in B nor in D are: $\{6, 8\}$ ■

(h) Translate the following English description into set builder notation: "The set of all pairs where the first element is a subset of the universe and the second element is the cardinality of the first element".

Solution:

$$\Big\{(S, |S|) \mid S \in \mathcal{P}(U)\Big\}$$

■

(i) Describe what the following set given in set builder notation contains in English:

$$\{S \subseteq U \mid |S| = 2, \forall a \in S, \forall b \in S : a \neq b \wedge a + b \text{ is even}\}$$

Solution:

"The set of all two element subsets S of the universe such that the sum of the two elements in the subset S is even." ■

(j) What is the power set of $C \cap D$?

Solution:

$C \cap D = \{0, 4\}$. The power set of that is: $\Big\{\{\}, \{0\}, \{4\}, \{0, 4\}\Big\}$ ■

(k) How many elements would there be in $U - C$?

Solution:

$U - C = \overline{C}$. We have $|C| = 4$, so $|\overline{C}| = |U| - |C| = 10 - 4 = 6$. So $U - C$ has 6 elements. ■

(l) How many elements would there be in the power set of $U - C$?

Solution:

The number of elements in the power set is 2^n where n is the size of the set whose power set we're calculating. $|U - C| = 6$, so the size of its power set is $2^6 = 64$. ■

Problem 5 (To and From Venn) 4 x 5

(a) Shade the Venn diagram in Figure 1 so that exactly the region represented by $A \setminus (B \cup C)$ is shaded.

Solution:

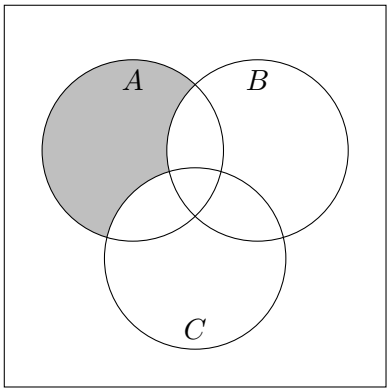


Figure 1: Venn diagram to shade.

■

(b) Shade the Venn diagram in Figure 2 so that exactly the region represented by $(A \Delta B) \setminus C$ is shaded.

Solution:

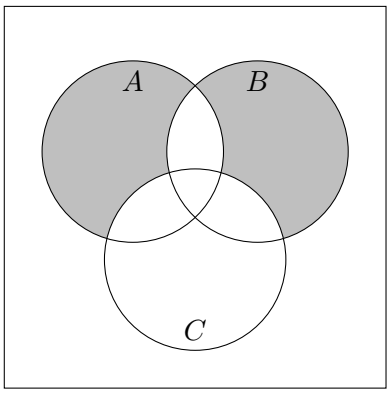


Figure 2: Venn diagram to shade.

■

(c) Given the shaded Venn diagram in Figure 3, give an expression using set operations that represents the shaded region.

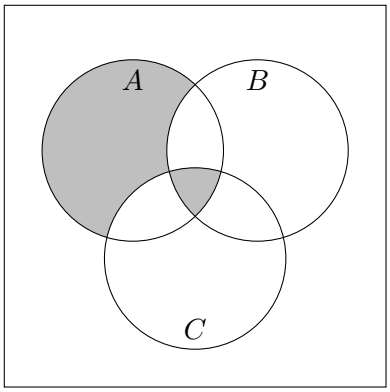


Figure 3: Shaded Venn diagram to find expression for.

Solution:

Expression: $\Big(A \setminus (B \cup C)\Big) \cup \Big(A \cap B \cap C\Big)$ ■

(d) Given the shaded Venn diagram in Figure 4, give an expression using set operations that represents the shaded region.

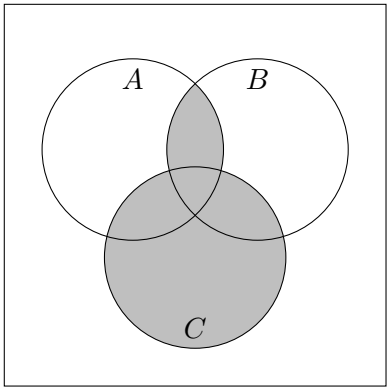


Figure 4: Shaded Venn diagram to find expression for.

Solution:

Expression: $C \cup (A \cap B)$ ■

Problem 6 (Miscellaneous Counting) 4 x 6

(a) 35 students from Northeastern university and 40 students from Boston university visit a museum. The museum functions with limited capacity and allows 20 students from one of the two schools to visit on that day. In how many ways can a team of 20 students visit the museum on that day?

Solution:

There are $\binom{35}{20}$ ways of choosing students from Northeastern and $\binom{40}{20}$ ways of choosing students from BU. Because we must choose a group from one school or the other, there are: $\binom{35}{20} + \binom{40}{20}$ total ways. Note that because either one school or the other sends a group of 20 students into the museum, we around counting the union of these two sets of combinations. (This is in contrast to how we often combine combination sets via the product rule. In those cases we're building a unique selection by choosing a combination from each set: the cartesian product). ■

(b) How many integers are between 0 and 999999 (including both) which don't have any digit repeated side-by-side. For example, 112 wouldn't be counted as the 1 digits are adjacent but 121 is a valid output.

Solution:

$$10 + 9^2 + 9^3 + 9^4 + 9^5 + 9^6$$

■

(c) How many ways can the letters in the word "MISCELLANEOUS" be rearranged?

Solution:

There are a total of 13 letters in the word. There are 2 S's, 2 L's, 2 E's, which are all the duplicate letters. We rearrange everything and then factor out the overcounting due to duplicate letters to get:

$$\frac{13!}{2! \cdot 2! \cdot 2!}$$

■

(d) There are 11 problems in this exam. How many ways can write this exam if your choices for each problem is to to leave it blank or to attempt something?

Solution:

Each problem can either be left blank or attempted. So there are $2 \times 2 \times \cdots \times 2$ possible ways to write this exam, which totals to 2^{11}. ■

Problem 7 (Racing) 6+7+7

A horse race has 12 horses.

(a) How many different ways can the podium be arranged?
(The podium has a spot for only the 1st, 2nd and 3rd place horse)

Solution:

$P(12,3) = \frac{12!}{9!} = 12 * 11 * 10$ ■

(b) How many different ways can the horses finish the race such that neither Grand Valor, Seabiscuit or Secretariat finish in first place?

Solution:

There are $P(12,12) = 12!$ ways that the horses finish without any restrictions.

- Given Secretariat wins the race, there are $P(11,11) = 11!$ ways the the other horses finish.
- Given Grand Valor wins the race, there are $P(11,11) = 11!$ ways the the other horses finish.
- Given Seabiscuit wins the race, there are $P(11,11) = 11!$ ways the the other horses finish.

We use the "difference/complement rule" to subtract the cardinality of these three sets from our total number of orderings. There are $12! - 3 * 11!$ ways that the horses finish such that none of three horses mentioned above finishes first. ■

(c) How many different ways can the horses finish the race such that one horse (Grand Valor) always beats both Seabiscuit and Secretariat?

Solution:

There are 12! total ways that all the horses finish. Lets split these finishes into 6 disjoint sets based on just how Grand Valor, Seabiscuit and Secretariat finish relative to each other:

- Grand Valor beats Seabiscuit who beats Secretariat
- Grand Valor beats Secretariat who beats Seabiscuit
- Seabiscuit beats Grand Valor who beats Secretariat
- Seabiscuit beats Secretariat who beats Grand Valor
- Secretariat beats Grand Valor who beats Seabiscuit
- Secretariat beats Seabiscuit who beats Grand Valor

(Just to check that this is all of the options, we remind ourselves that there are $3! = 6$ ways of ordering 3 horses).

Each of these groups has the *same* amount of the 12! total ways the horses finish within it. Only the first 2 of the 6 meet our requirements though. So there are $12! * 2/6$ unique finishes in which Grand Valor beats these two other horses. ■

Problem 8 (Pigeons at Ruggles) 6+7+7

There are 35 pigeons in a flock that roosts on the top of Ruggles.

(a) What is the maximum number of nests such that there is a nest with a minimum of three pigeons?

Solution:

Solving the pigeonhole problem we get that

$$3 = \left\lceil \frac{35}{k} \right\rceil$$

So we need 17 nests to have a nest with at least three pigeons on it, since any greater number would result in 2 pigeons per nest. ■

(b) If there are 18 nests on top of Ruggles for the 35 identical pigeons, how many different ways can the pigeons be arranged?

Solution:

There are 18 piles and 35 items, this is a balls and bins problem. $\binom{18+35-1}{18-1}$ ways of arranging the pigeons. ■

(c) Two of the 35 pigeons have paired off and now will share one of the 18 nests with only themselves, eliminating the nest for the other pigeons, how many ways can the remaining identical pigeons be arranged in the remaining nests?

Solution:

Once the paired off pigeons use up one of the nests, there are 17 piles and 33 items, this is a balls and bins problem. $\binom{17+33-1}{17-1}$ ways of arranging the pigeons. Now we need to count the number of ways the single nest can be chosen, so the result is $\binom{18}{1}\binom{17+33-1}{17-1}$. ■

Problem 9 (Cards) **5 x 8**

There are a total of 36 cards. Each card has a number, from 1 to 9 as well as a color from: Cyan, Magenta, Yellow or Black such that every possible number+color combinations exists exactly once. So, the cards can be labelled using the elements of: $\{1, 2, \ldots, 9\} \times \{\text{Cyan, Magenta, Yellow, Black}\}$. How many ways can:

(a) these 36 cards be separated into 6 piles named A, B, C, D, E, F such that each pile has an equal number of cards?

Solution:

Equal distribution would mean 6 cards per pile. Choose 6 cards for A from the 36 cards. From the remaining 30, choose 6, and so on.

$$\binom{36}{6} \cdot \binom{30}{6} \cdot \binom{24}{6} \cdot \binom{18}{6} \cdot \binom{12}{6} \cdot \binom{6}{6}$$

■

(b) these 36 cards be separated into 6 unlabeled, indistinguishable piles such that each pile has an equal number of cards?

Solution:

We first separate into 6 piles labelled A to F. Then we factor out the overcounting due to all piles being unlabelled and indistinguishable. This factoring out of overcounting is done by dividing by 6!. So we get:

$$\frac{1}{6!} \cdot \binom{36}{6} \cdot \binom{30}{6} \cdot \binom{24}{6} \cdot \binom{18}{6} \cdot \binom{12}{6} \cdot \binom{6}{6}$$

■

(c) we arrange the odd numbered cards in order such that all same color cards are together?

Solution:

Only odd numbered cards are there, so for each color we have the 5 numbers 1, 3, 5, 7, 9. Each color is together, so the 5 cards of each color can be arranged in 5! ways. The order of colors is also flexible, which can be arranged in 4! ways. Together we get:

$$5! \cdot 5! \cdot 5! \cdot 5! \cdot 4!$$

■

(d) six cards be chosen such that there are exactly two cards from any three separate colors?

Solution:

Choose the three colors, in $\binom{4}{3}$ ways. For each of the chosen colors, choose 2 out of the 9 numbers in $\binom{9}{2}$ ways. In total:

$$\binom{4}{3} \cdot \binom{9}{2} \cdot \binom{9}{2} \cdot \binom{9}{2}$$

■

(e) nine cards be chosen such that either, they are all the same color or there are three cards of three different colors?

Solution:

If all nine cards are the same color, then we just need to choose one of the four colors. If there are three cards of three different colors, then we need to choose which three colors and then choose three numbers for the chosen three colors. In the end, sum rule is used to combine and get the final count:

$$\binom{4}{1} + \binom{4}{3} \cdot \binom{9}{3} \cdot \binom{9}{3} \cdot \binom{9}{3}$$

■

Problem 10 (What's the chance?) 5+10+10+10

There are three bins named A, B, and C. Bin A contains 3 white and 4 red balls. Bin B contains 5 white and 5 red balls. Bin C contains 6 white and 2 red balls.

(a) What is the probability of picking 3 white balls from bin B, if they are picked one after the other, without replacement (without being put back in before next ball is picked)?

Solution:

Bin B has 5 white and 5 red balls.

$$Pr(WWW) = \frac{5 \cdot 4 \cdot 3}{10 \cdot 9 \cdot 8}$$

■

(b) What is the joint probability table of picking one ball from bin A and one from bin B?

Solution:

	$A =$ White	$A =$ Red
$B =$ White	$\frac{3}{7} \cdot \frac{5}{10}$	$\frac{4}{7} \cdot \frac{5}{10}$
$B =$ Red	$\frac{3}{7} \cdot \frac{5}{10}$	$\frac{4}{7} \cdot \frac{5}{10}$

Table 2: Joint probability table of picking a single ball from A and B

■

(c) If one ball is picked from each of the 3 bins, then what is the probability that there are two white and one red ball?

Solution:

We can get two white and one red ball in three ways based on which bin the red ball came from. Writing the color of the ball from bins A, B, and C into a triple (A's color, B's color, C's color), we could have: (R, W, W) or (W, R, W) or (W, W, R).

$$\begin{aligned} Pr(\text{one red, two whites}) &= Pr\big((R,W,W)\big) + Pr\big((W,R,W)\big) + Pr\big((W,W,R)\big) \\ &= \frac{4}{7} \cdot \frac{5}{10} \cdot \frac{6}{8} + \frac{3}{7} \cdot \frac{5}{10} \cdot \frac{6}{8} + \frac{3}{7} \cdot \frac{5}{10} \cdot \frac{2}{8} \end{aligned}$$

■

(d) What is the probability that two balls are picked from each of the three bins and all the first balls are white and all the second balls drawn are red?

Solution:

We want W then R for bin A, W then R for bin B, W then R for bin C. These are independent events, so we can multiply the probabilities computed for each bin separately

$$\begin{aligned} & Pr(A \text{ has W then R}) \cdot Pr(B \text{ has W then R}) \cdot Pr(C \text{ has W then R}) \\ &= \frac{3}{7} \cdot \frac{4}{6} \cdot \frac{5}{10} \cdot \frac{5}{9} \cdot \frac{6}{8} \cdot \frac{2}{7} \end{aligned}$$

■

Problem 11 (That's unlikely) 4 x 5

You roll a 20-sided dice with faces numbered from 1 to 20. What is the probability of the following events:

(a) The dice roll is 1.

Solution:

$$Pr(\text{roll } = 1) = \frac{1}{20}$$

■

(b) The dice roll is an even number.

Solution:

$$Pr(\text{roll is even}) = Pr(\text{roll} \in \{2, 4, 6, \ldots, 18, 20\}) = \frac{10}{20} = \frac{1}{2}$$

■

(c) The dice roll is greater than 15.

Solution:

$$Pr(\text{roll is great than } 15) = Pr(\text{roll} \in \{16, 17, 18, 19, 20\}) = \frac{5}{20} = \frac{1}{4}$$

■

(d) The dice is rolled twice and both rolls are the same.

Solution:

Let's assume that the first roll came out to be some value x. We want:

$$Pr(\text{2nd roll} = \text{first roll } x) = Pr(\text{2nd roll} = x) = \frac{1}{20}$$

■