CS1800 Discrete Structures

Solutions to Exam Practice 1

Strange & Varma *Date: September 27, 2026*

- **Actual vs. Practice:** The primary difference in the actual exam vs. this practice exam is number of subparts per problem. You can expect each of the 7 problems to only have 1 (or maybe 2) subparts instead of the multiple in this exam. The relative proportion of points will be similar in the actual exam, but the total will be different. The instructions below will also be the same in the actual exam.

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID in the blank space provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **7** problems totaling **278** points. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. Show your work, as partial credit may be given. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring one 8.5x11–inch cheat sheet, with anything written/typed on it, only one side of the sheet can be used. No other materials or devices are permitted. No collaboration of any kind is permitted.
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	Total
Points	40	36	40	64	20	28	40	268

Best of luck!

Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Annihiltor/Domination Laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotence Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Complement/Negation Laws	$p \wedge \neg p \equiv F$ $p \vee \neg p \equiv T$
Double Negation Law	$\neg\neg p \equiv p$
Absorption Laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Table 1: Laws of Boolean Algebra for reference.

Problem 1 (TT $\longrightarrow$ Circuit) 10 x4

(a) Draw the corresponding circuit for the Truth Table in Table 2. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Output
0	0	1	1	1
0	1	0	1	1
1	0	1	0	1
1	1	0	0	1

Table 2

Solution:

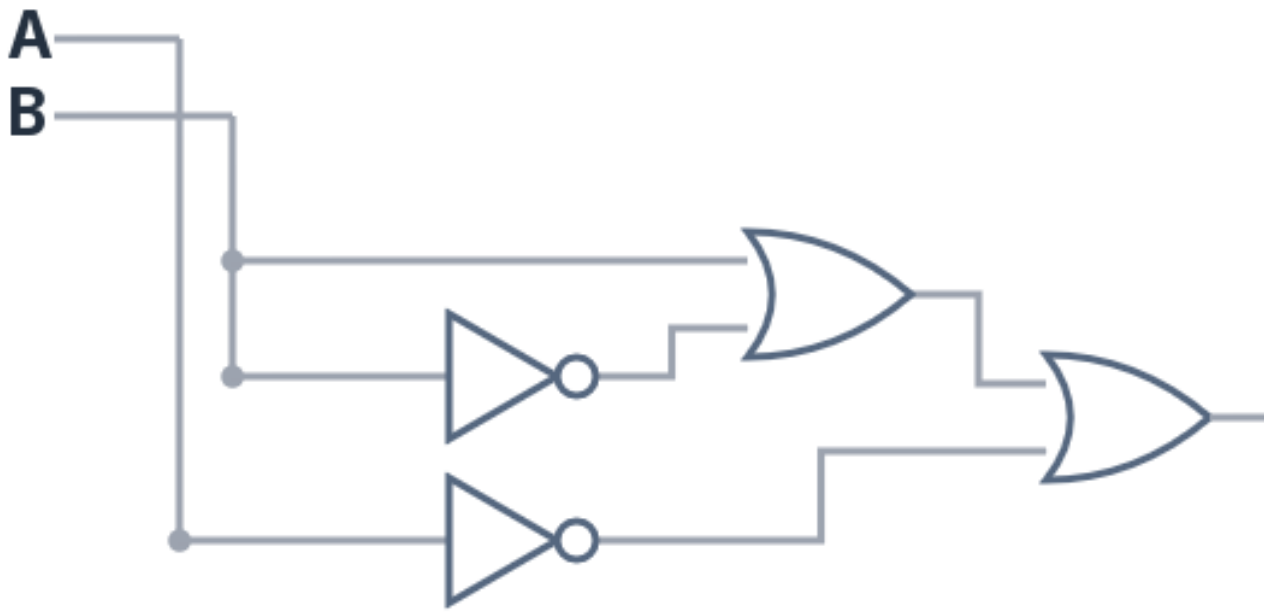


Figure 1: Circuit representing the Truth Table in Table 2

■

(b) Draw the corresponding circuit for the Truth Table in Table 3. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Z	Output
0	0	1	1	1	0
0	1	0	1	1	0
1	0	1	0	1	1
1	1	0	0	0	0

Table 3

Solution:

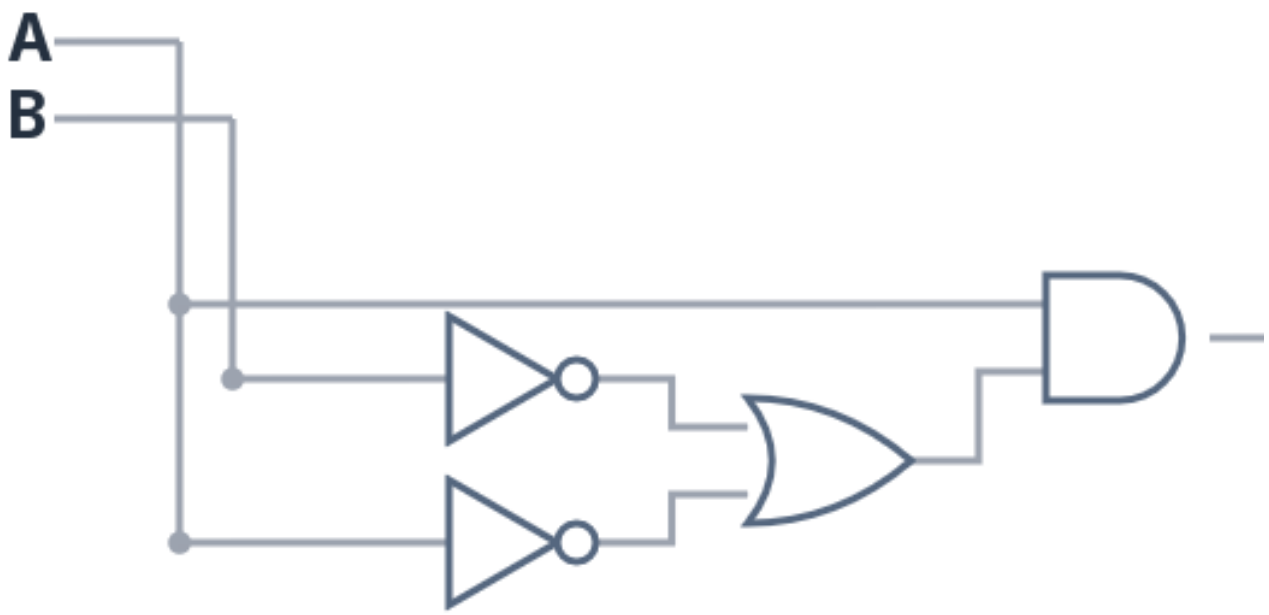


Figure 2: Circuit representing the Truth Table in Table 3

■

(c) Draw the corresponding circuit for the Truth Table in Table 4. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Z	Output
0	0	1	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	1	0	1	1	1

Table 4

Solution:

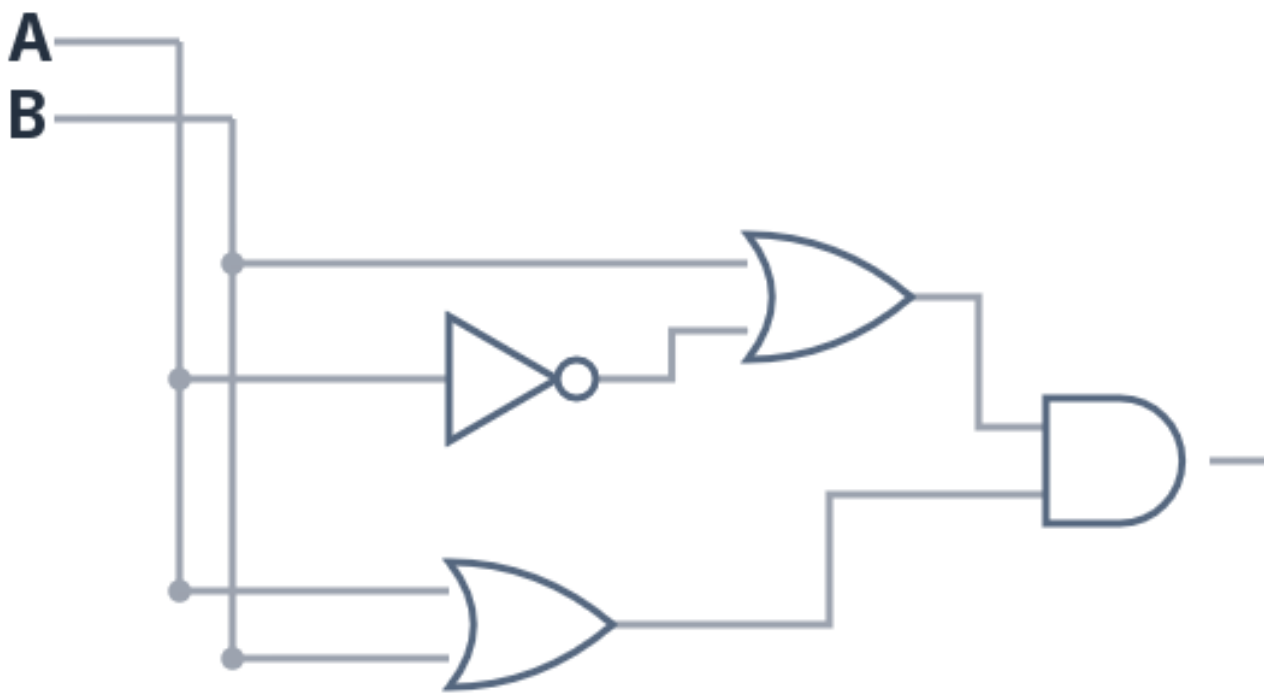


Figure 3: Circuit representing the Truth Table in Table 4

■

(d) Draw the corresponding circuit for the Truth Table in Table 5. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

A	B	X	Y	Output
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

Table 5

Solution:

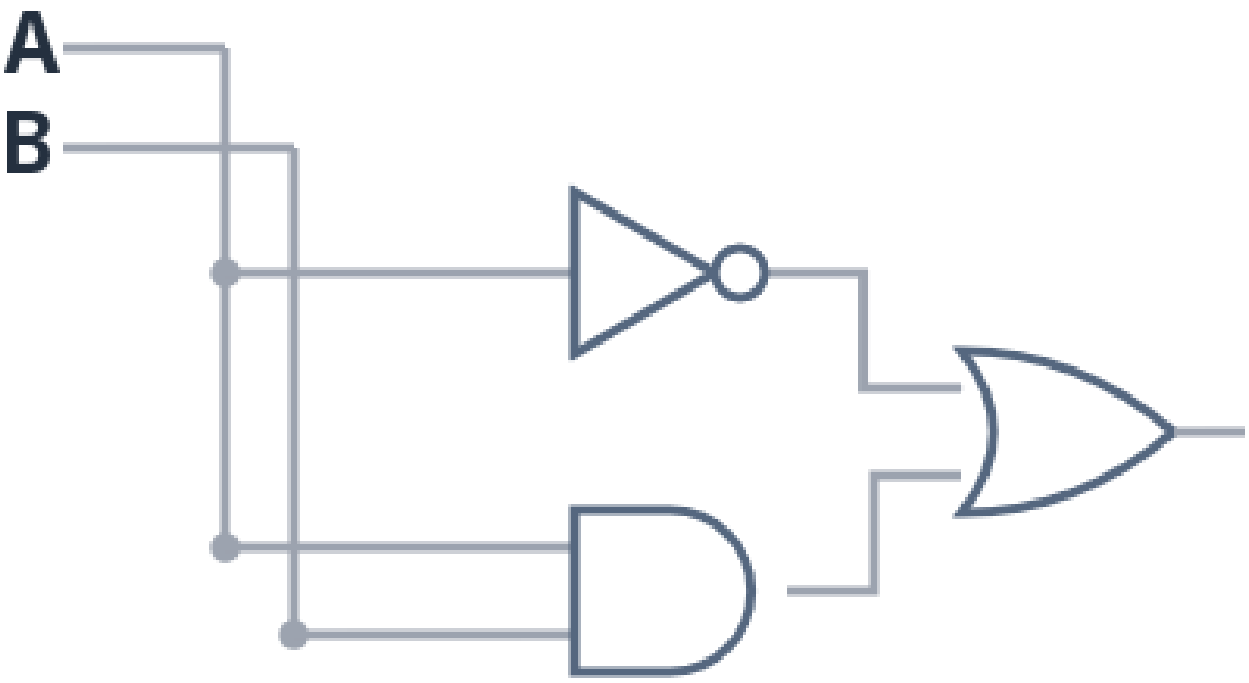


Figure 4: Circuit representing the Truth Table in Table 5

■

Problem 2 (Circuit $\longrightarrow$ Logical Expression) (5+2+2) x4

(a) Convert the circuit in Figure 5 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

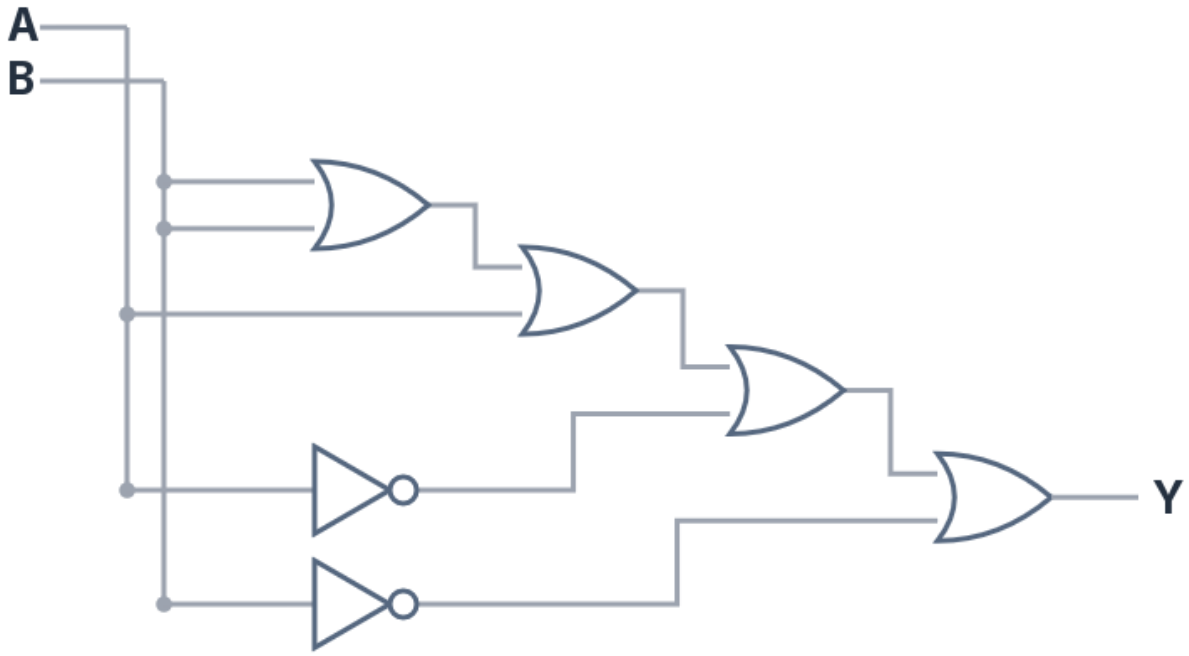


Figure 5

Solution:

Boolean Formula: $(b \vee b \vee a \vee \neg a \vee \neg b)$

This is equivalent to T, so any input will result in a True output and None which will result in a False output. ■

(b) Convert the circuit in Figure 6 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

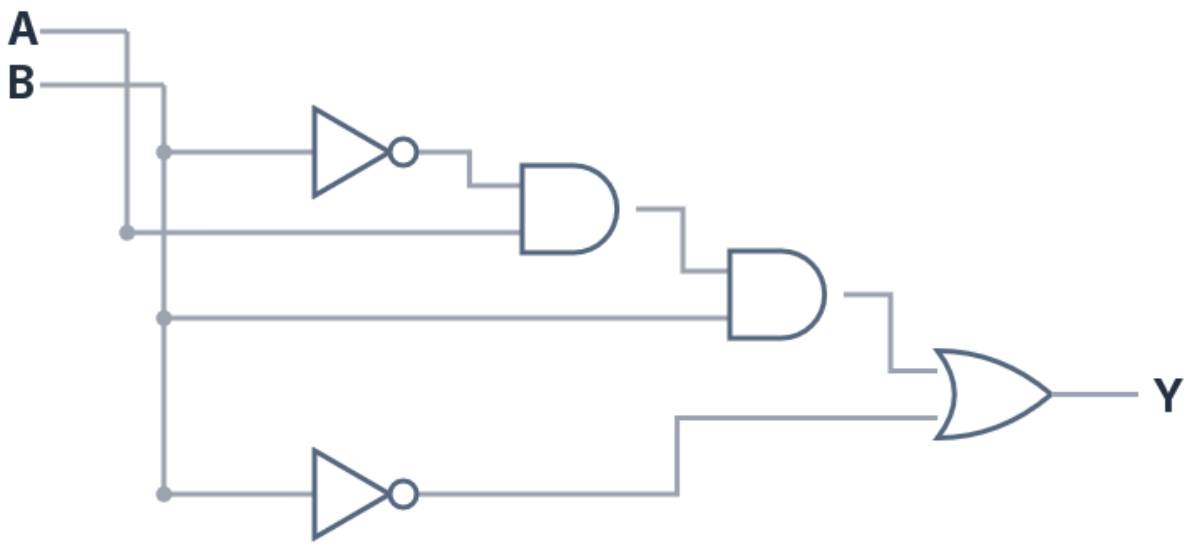


Figure 6

Solution:

Boolean Formula: $(\neg b \wedge a \wedge b) \vee \neg b$

Setting $b = F$ allows the output to be True due to the $\vee \neg b$ part. Since the part inside the parenthesis has $\neg b \wedge b$ it is always false. So, to get the output to be False, we only need to set $b = T$. ■

(c) Convert the circuit in Figure 7 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

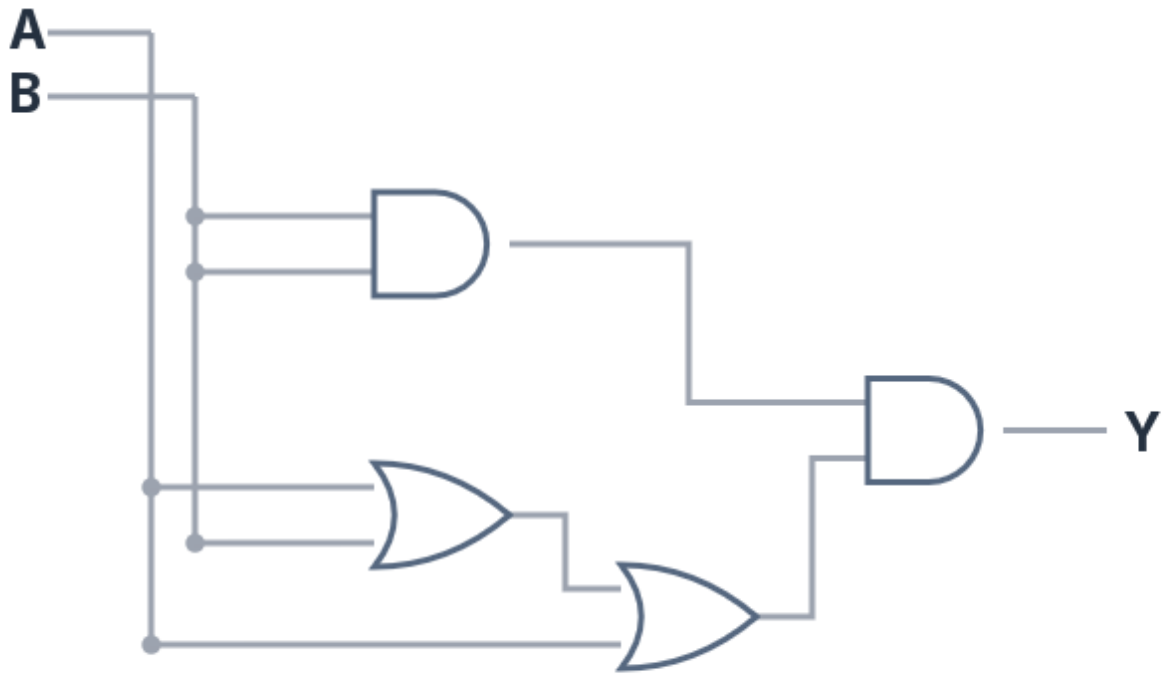


Figure 7

Solution:

Boolean Formula: $b \wedge b \wedge (a \vee b \vee a)$

To get the output to be True, we need $b = T$ since the expression simplifies to just b via applications of absorption and idempotent laws. If $b = F$, then the output is False. ■

(d) Convert the circuit in Figure 8 to a logical expression with boolean variables a, b.

Provide a value for a, b such that the output is True and another value for a, b such that the output is False. If no input values work, then say "None".

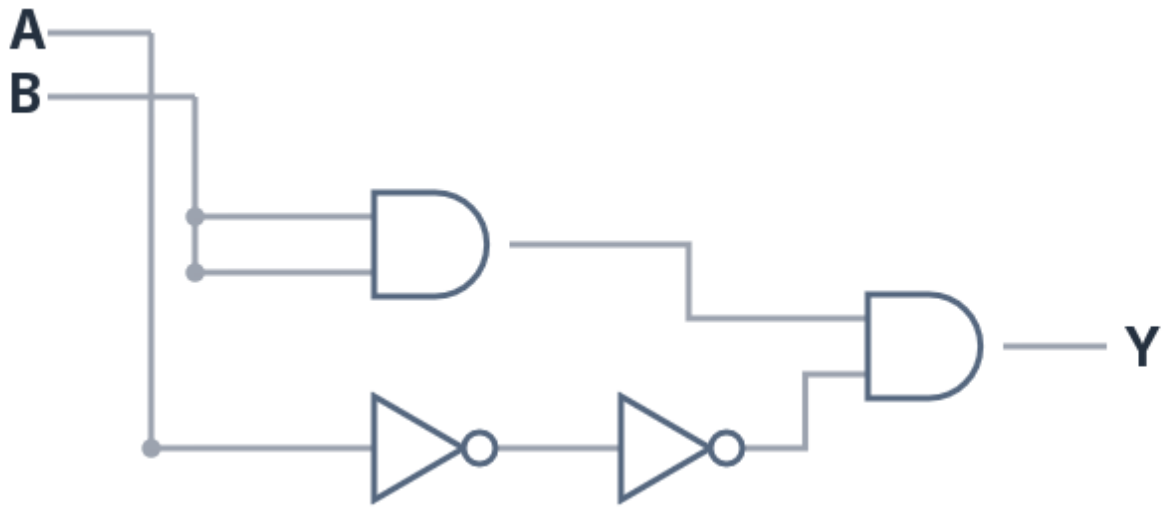


Figure 8

Solution:

Boolean Formula: $(b \wedge b \wedge \neg\neg a)$

This is equivalent to $b \wedge a$, so $a = b = T$ ensures output is True while anything else will give a False output. ■

Problem 3 (Logical Expression Equivalence via TT) 10 x4

(a) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $(B \vee A) \wedge \neg B \wedge \neg A$ Statement 2: F

Solution:

a	b	$b \vee a$	$\neg b$	$\neg a$	$(b \vee a) \wedge \neg b \wedge \neg a \equiv F$
0	0	0	1	1	0
0	1	1	0	1	0
1	0	1	1	0	0
1	1	1	0	0	0

Table 6: Truth Table for: $(B \vee A) \wedge \neg B \wedge \neg A$. The last column matches statement 2: F.

■

(b) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $C \vee \neg (C \vee A)$ Statement 2: $C \vee \neg A$

Solution:

A	C	$C \vee A$	$\neg (C \vee A)$	$C \vee \neg (C \vee A)$
0	0	0	1	1
0	1	1	0	1
1	0	1	0	0
1	1	1	0	1

a	c	$\neg a$	$c \vee \neg a$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

Table 7: Truth Tables for $C \vee \neg (C \vee A)$ and $C \vee \neg A$. The last columns match.

■

(c) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $(A \wedge (B \vee \neg A)) \vee \neg C$ Statement 2: $(A \wedge B) \vee \neg C$

Solution:

a	b	c	$\neg a$	$b \vee \neg a$	$a \wedge (b \vee \neg a)$	$\neg c$	$(a \wedge (b \vee \neg a)) \vee \neg c$
0	0	0	1	1	0	1	1
0	0	1	1	1	0	0	0
0	1	0	1	1	0	1	1
0	1	1	1	1	0	0	0
1	0	0	0	0	0	1	1
1	0	1	0	0	0	0	0
1	1	0	0	1	1	1	1
1	1	1	0	1	1	0	1

a	b	c	$a \wedge b$	$\neg c$	$(a \wedge b) \vee \neg c$
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	1	0	1

Table 8: Truth Tables for $(A \wedge (B \vee \neg A)) \vee \neg C$ and $(A \wedge B) \vee \neg C$. The last columns match.

■

(d) Are the following statements logically equivalent? If so, show their truth table; if not, show the inputs on which they do not agree.

Statement 1: $A \vee (\neg B \vee A)$ Statement 2: $B \vee A$

Solution:

These two statements are not equivalent.

Consider $a = F$ and $b = F$, then $A \vee (\neg B \vee A)$ is T while $B \vee A$ is F. ■

Problem 4 (Boolean Formula ≡ Boolean Formula) 16 x4

(a) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

Solution:

$$C \vee (\neg B \wedge \neg A \wedge (A \vee A \vee B))$$

$$= C \vee (\neg (A \vee B) \wedge (A \vee A \vee B)) \quad \text{DeMorgan's}$$

$$= C \vee (\neg (A \vee B) \wedge (A \vee B)) \quad \text{Idempotent}$$

$$= C \vee \mathrm{F} \quad \text{Complement}$$

$$= C \quad \text{Identity}$$

■

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(b) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

Solution:

$$\neg B \vee \neg A \vee (C \wedge \neg A) \vee \neg B \vee C \vee \neg A$$

$$= \neg B \vee \neg A \vee \neg B \vee C \vee \neg A \qquad \text{Absorption}$$

$$= \neg B \vee \neg A \vee C \vee \neg A \qquad \text{Idempotent}$$

$$= \neg B \vee \neg A \vee C \qquad \text{Idempotent}$$

$$= \neg (B \wedge A) \vee C \qquad \text{DeMorgan's}$$

■

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(c) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

Solution:

$$((B \vee C) \wedge (B \vee \neg A \vee C)) \vee \neg\neg\neg A$$

$$= ((B \vee C) \wedge (B \vee \neg A \vee C)) \vee \neg A \qquad \text{Double Negation}$$

$$= B \vee (C \wedge (\neg A \vee C)) \vee \neg A \qquad \text{Distributive}$$

$$= B \vee C \vee \neg A \qquad \text{Absorption}$$

■

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

(d) In this simplfication, write beside each step the rule being used in the proof of equivalence or fill in the missing step given the rule. *Note: It is not necessary to perform commutative and associative law applicaiton in separate steps.*

Solution:

$$\neg\neg\left(A \vee \neg A \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right)$$

$$= \left(A \vee \neg A \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right) \qquad \text{Double Negation}$$

$$= \left(\mathrm{T} \vee B\right) \wedge \left(\neg\left(A \wedge C\right) \vee \neg\left(A \vee \neg C\right)\right) \qquad \text{Complement}$$

$$= \left(\mathrm{T} \vee B\right) \wedge \neg\left(A \wedge C \wedge \left(A \vee \neg C\right)\right) \qquad \text{DeMorgan's}$$

$$= \left(\mathrm{T} \vee B\right) \wedge \neg\left(A \wedge C\right) \qquad \text{Absorption}$$

$$= \mathrm{T} \wedge \neg\left(A \wedge C\right) \qquad \text{Domination}$$

$$= \neg\left(A \wedge C\right) \qquad \text{Identity}$$

■

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

Problem 5 (Implications) (6 + 4) x2

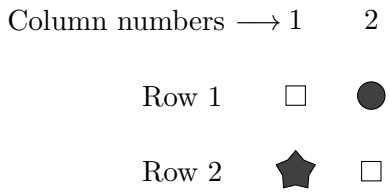


Figure 9: Universe

For each of the following statements, provide its contrapositive, converse, inverse (all in English, no logical operators required). For each of the 4 statements, determine its truth value based on Figure 9.

(a) "If the shape is on the diagonal, then it is a clear square."
[***Hint**: The diagonal has row number equal to column number.*]

Solution:

Original: *True.*

Contrapositive: If the shape is not a clear square then it is not on the diagonal.
True.

Converse: If the shape is a clear square then it is on the diagonal.
True.

Inverse: If the shape is not on the diagonal then it is not a clear square.
True. ■

(b) "If the shape is shaded then it is not a square."

Solution:

Original: *True.*

Contrapositive: If the shape is a square then it is not shaded.
True.

Converse: If the shape is not a square then it is shaded.
True.

Inverse: If the shape is not shaded then it is a square.
True. ■

Problem 6 (Predicates and Quantifiers) (5+2) x4

In the following, you can use the predicates $Square(x)$, $Circle(x)$, $Star(x)$ to denote whether a given shape x is a square, circle or star respectively. Similarly, $Shaded(x)$ and $Clear(x)$ are predicates denoting that a shape has been shaded or is clear (unshaded). $Row(x)$ is the row number (from 1, 2, 3) of a given shape x. $Col(x)$ is the column number (from 1, 2, 3, 4) of a given shape x.

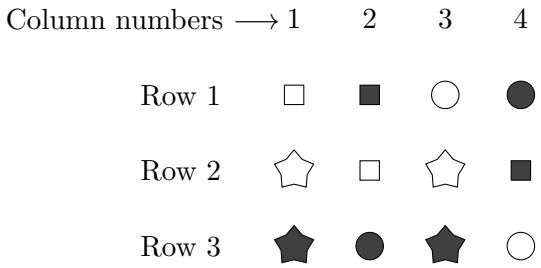


Figure 10: Universe

For the following statements, if they are in English, translate to a Boolean formula and vice versa. For each statement, also mention if it is True/False. Provide a counterexample if the statement is False.

(a) "If the shape is a Star, then it is shaded."

Solution:

$Star(x) \implies Shaded(x)$

False: there are stars which are clear, in Row 2 columns 1 and 3. ■

(b) $\forall x : Clear(x) \implies Circle(x)$

Solution:

"Given any shape, if it is clear, then it is a circle."

False: there are clear squares and stars, in row 1, column 1 and in the first 3 columns of row 2. ■

(c) "Every clear square has a shaded shape below it."

Solution:

$\forall x, \exists y : Clear(x) \land Square(x) \implies Shaded(y) \land \Big(Row(x) < Row(y) \Big)$

True: The two clear squares are in Row 1, Column 1 and Row 2, Column 2. The first two shaded shapes in Row 3 satisfy the requirement for a shaded shape below the clear squares. ■

(d) $\forall x, \exists y : Square(x) \land Clear(x) \implies Square(y) \land Shaded(y) \land \Big(Col(x) < Col(y) \Big)$

Solution:

"Given any clear square, there is a shaded square to the right of it."

True: In row 1, columns 1 and 2 have a clear and shaded square respectively. Similarly, in row 2, columns 2 and 4 have a clear and shaded square respectively. ■

Problem 7 (Number Conversions) 5 x8

(a) `oct2dec`: Convert 1234_8 to decimal; show all steps.

Solution:

$$\begin{aligned} 1234_8 &= 1 \cdot 8^3 + 2 \cdot 8^2 + 3 \cdot 8^1 + 4 \cdot 8^0 \\ &= 1 \cdot 512 + 2 \cdot 64 + 3 \cdot 8 + 4 \cdot 1 \\ &= 512 + 128 + 24 + 4 \\ &= 668 \end{aligned}$$

■

(b) `dec2bin`: Convert 1234_{10} to base 2 using the division method; show all steps.

Solution:

We'll track quotient q and remainder r on division by 2 in the form (q, r). The remainders will give us the base 2 number from right to left.

$$\begin{aligned} 1234 &\longrightarrow (617, 0) \\ &\longrightarrow (308, 1) \\ &\longrightarrow (154, 0) \\ &\longrightarrow (77, 0) \\ &\longrightarrow (38, 1) \\ &\longrightarrow (19, 0) \\ &\longrightarrow (9, 1) \\ &\longrightarrow (4, 1) \\ &\longrightarrow (2, 0) \\ &\longrightarrow (1, 0) \\ &\longrightarrow (0, 1) \end{aligned}$$

$1234_{10} = 10011010010_2$ ■

(c) `hex2dec`: Convert AB_{16} to decimal; show all steps.

Solution:

$$\begin{aligned} AB_{16} &= A \cdot 16^1 + B \cdot 16^0 \\ &= 10 \cdot 16 + 11 \cdot 1 \\ &= 160 + 11 \\ &= 171 \end{aligned}$$

■

(d) `dec2oct`: Convert 89_{10} to base 8 using the division method; show all steps.

Solution:

We'll track quotient q and remainder r on division by 8 in the form (q, r). The remainders will give us the base 8 number from right to left.

$$\begin{aligned} 89 &\longrightarrow (11, 1) \\ &\longrightarrow (1, 3) \\ &\longrightarrow (0, 1) \end{aligned}$$

$89_{10} = 131_8$ ■

(e) `qua2dec`: Convert 3333_4 to decimal; show all steps.

Solution:

$$\begin{aligned} 3333_4 &= 3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 \\ &= 3 \cdot 64 + 3 \cdot 16 + 3 \cdot 4 + 3 \cdot 1 \\ &= 192 + 48 + 12 + 3 \\ &= 255 \end{aligned}$$

■

(f) `dec2qua`: Convert 456_{10} to base 4 using the division method; show all steps.

Solution:

We'll track quotient q and remainder r on division by 4 in the form (q, r). The remainders will give us the base 4 number from right to left.

$$\begin{aligned} 456 &\longrightarrow (114, 0) \\ &\longrightarrow (28, 2) \\ &\longrightarrow (7, 0) \\ &\longrightarrow (1, 3) \\ &\longrightarrow (0, 1) \end{aligned}$$

$456_{10} = 13020_4$ ■

(g) `bin2dec`: Convert 110101001_2 to decimal; show all steps.

Solution:

$$\begin{aligned}
110101001_2 &= 1 \cdot 2^8 + 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\
&= 1 \cdot 256 + 1 \cdot 128 + 0 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 \\
&= 256 + 128 + 32 + 8 + 1 \\
&= 425
\end{aligned}$$

■

(h) `dec2bin`: Convert 456_{10} to base 2 using the division method; show all steps.

Solution:

We'll track quotient q and remainder r on division by 2 in the form (q, r). The remainders will give us the base 2 number from right to left.

$$\begin{aligned}
456 &\longrightarrow (228, 0) \\
&\longrightarrow (114, 0) \\
&\longrightarrow (57, 0) \\
&\longrightarrow (28, 1) \\
&\longrightarrow (14, 0) \\
&\longrightarrow (7, 0) \\
&\longrightarrow (3, 1) \\
&\longrightarrow (1, 1) \\
&\longrightarrow (0, 1)
\end{aligned}$$

$456_{10} = 111001000_2$ ■