CS1800 Discrete Structures

Solutions to Exam: 2

Strange & Varma *Date: November 4, 2026*

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID and Name in the blank spaces provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **7 problems** totaling **110 points**. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. **Show your work**, as partial credit may be given for correct approach even without the final answer. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring **one US letter (8.5x11–inch)** cheat sheet, with anything written/typed on it; **only one side** of the sheet can be used. **No other materials or devices are permitted. No collaboration of any kind is permitted.**
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	Total
Points	12	15	18	10	20	15	20	110

Best of luck!

Table 1: Laws of Set Operations. A, B, C are arbitrary sets; U is the universal set; $\emptyset$ is the empty set.

Commutative Laws	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative Laws	$(A \cap B) \cap C \equiv A \cap (B \cap C)$ $(A \cup B) \cup C \equiv A \cup (B \cup C)$
Distributive Laws	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Identity Laws	$A \cap U \equiv A$ $A \cup \emptyset \equiv A$
Annihiltor/Domination Laws	$A \cap \emptyset \equiv \emptyset$ $A \cup U \equiv U$
Idempotence Laws	$A \cap A \equiv A$ $A \cup A \equiv A$
Complement/Negation Laws	$A \cap A^C \equiv \emptyset$ $A \cup A^C \equiv U$
Double Negation Law	$\left(A^C\right)^C \equiv A$
Absorption Laws	$A \cap (A \cup B) \equiv A$ $A \cup (A \cap B) \equiv A$
De Morgan's Laws	$\overline{(A \cap B)} \equiv \overline{A} \cup \overline{B}$ $(A \cup B)^C \equiv A^C \cap B^C$

Problem 1 (Laws of Set Operations) 12

In this simplification of the set expression involving sets A, B, and C, on each line, write the law of set operations being used or fill in the missing step given the name of the law. *Note: It is not necessary to perform commutative and associative law application in separate steps; they do not need to be mentioned.*

Solution:

$$(A \cup C) \cap (C \cup B) \cap \overline{\overline{C} \cap B}$$

$$= (A \cup C) \cap (C \cup B) \cap \left(\overline{\overline{C}} \cup \overline{B}\right) \quad \text{DeMorgan's}$$

$$= (A \cup C) \cap (C \cup B) \cap \left(C \cup \overline{B}\right) \quad \text{Double Negation}$$

$$= C \cup \left(A \cap B \cap \overline{B}\right) \quad \text{Distributive}$$

$$= C \cup (A \cap \emptyset) \quad \text{Complement}$$

$$= C \cup \emptyset \quad \text{Domination}$$

$$= C \quad \text{Identity}$$

■

Problem 2 (4-bit Two's Complement) 5+5+5

In this problem, all two's complement is in 4-bit two's complement notation.

(a) Convert -4 into 4-bit two's complement notation.

Solution:

$(4)_{10} = (0100)_2$, so $-4 = \overline{0100} + 1 = 1011 + 1 = 1100$ ■

(b) The numbers $x = 1001$ and $y = 1010$ are in two's complement notation. Add these, verify whether there is overflow or not.

Solution:

Carry	1	0	0	0	
		1	0	0	1
+		1	0	1	0
	1	0	0	1	1

So the result is $x + y = 0011$. The two numbers were negative but the result is positive, so there is overflow.

■

(c) Convert the number $z = 1101$ from two's complement into decimal.

Solution:

Converting to decimal is done via the same process; complement, then add 1:
$1101 \longrightarrow 0010 \longrightarrow -(0011)_2 \longrightarrow -(3)_{10}$

Alternatively, you can subtract 1 and then complement:
$1101 \longrightarrow 1100 \longrightarrow -(0011)_2 \longrightarrow -(3)_{10}$

■

Problem 3 (Sets) 2+3+2+2+2+2+3+2

In this problem, the universe is the set $U = \{1, 2, 3, 4, 5\}$. We also have the sets $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$.

(a) True/False: $3 \in A \cap B$.

Solution:

True. ■

(b) Write the set $\{x^2 \mid x \in B\} \setminus C$ in Roster notation.

Solution:

$\{9, 16\}$ ■

(c) What is the cardinality of: $C \times D$?

Solution:

$2 \times 3 = 6$ ■

(d) True/False: $(4, 5) \in C \times D$.

Solution:

True. ■

(e) True/False: $\{(5,3)\} \subset \overline{C \times D}$.

Solution:

True. ■

(f) True/False: $\{1,5\} \in \mathcal{P}(D)$.

Solution:

True. ■

(g) True/False: $\{\{1\},\{5\}\} \subseteq \mathcal{P}(D)$.

Solution:

True. ■

(h) What is $|\mathcal{P}(U-C)|$?

Solution:

2^3 ■

Problem 4 (To and From Venn) 5 + 5

(a) Shade the Venn diagram in Figure 1 so that exactly the region represented by $(A \setminus B)\Delta C$ is shaded.

Solution:

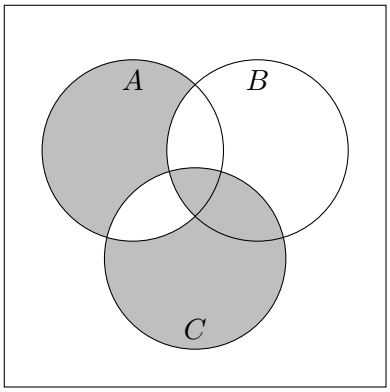


Figure 1: Venn diagram to shade.

■

(b) Given the shaded Venn diagram in Figure 2, give an expression using set operations that represents the shaded region.

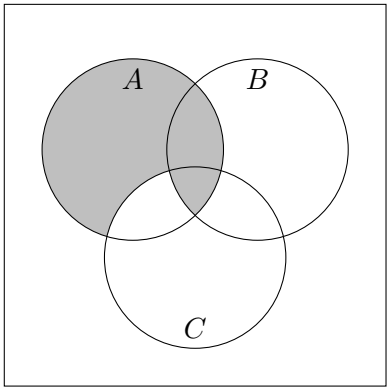


Figure 2: Shaded Venn diagram to find expression for.

Solution:

Expression: $(A \setminus C) \cup (A \cap B \cap C)$ ■

Problem 5 (Counting) 6 + 7 + 7

(a) How many ways can the letters in the word "COMMUTATIVE" be rearranged?

Solution:

There are a total of 11 letters in the word.

There are two M's and 2 T's, which are all the duplicate letters. We rearrange everything and then factor out the overcounting due to duplicate letters to get:

$$\frac{11!}{2! \cdot 2!}$$

■

(b) How many integers between 10 and 999 (both inclusive) exist which end with an even digit?

Solution:

The integer could have 2 or 3 digits. Since the integer must end in an even digit, the last digit could be any of 5 options. The leftmost digit must be non-zero (9 options). Everything else could be any of the 10 digits. Putting it all together, we get:

$$9 \cdot 5 + 9 \cdot 10 \cdot 5$$

■

(c) How many ways can 30 balls be placed into 4 bins, which are named A, B, C, and D, if the bin named A must have at least 2 balls in it?

Solution:

We first set aside $k = 2$ balls to be placed into bin A. The remaining $30 - k$ balls can be placed into the 4 bins however. This is a stars and bars problem, with $30 - k$ stars to be arranged with $4 - 1$ bars. This gives us:

$$\binom{30 - k + (4 - 1)}{30 - k} = \binom{30 - 2 + (4 - 1)}{30 - 2}$$

■

Problem 6 (Probability) 5+10

(a) If a program generates all 7-bit strings with equal probability, what is the probability what the generated string has 4 ones and 3 zeros?

Solution:

Number of ways the strings we want show up can be counted by deciding the positions where the ones show up, the remaining positions out of the 7 total positions will automatically be zeros. This will be the event space size. We divide by 2^7, the total number of 7-bit strings.

$$\frac{\binom{7}{4}}{2^7}$$

■

(b) If two 8-sided dice, A and B, are rolled, what is the probability that the roll of dice A is less than or equal to 4 and the sum of the two rolls is 6?

Solution:

The probability that dice A is less than or equal to 4 is:

$$Pr(\text{roll } \in \{1,2,3,4\}) = \frac{4}{8} = \frac{1}{2}$$

For the sum to be equal to 6, we need the value of dice B to be paired as:

$$(1,5),(2,4),(3,5),(4,2)$$

Here, the pairs are in the form (value of A, value of B).

In other words, once the roll of dice A is decided, there is exactly one possible value for dice B such that the sum is what we want. The probability of that happening is precisely: $1/8$.

Putting it all together, the overall probability is:

$$\frac{1}{2} \cdot \frac{1}{8}$$

. ■

USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.

Problem 7 (Bin A and Bin B) 5+5+10

There are two bins, A and B. Bin A has 3 white and 2 red balls. Bin B has 4 white and 3 red balls.

(a) What is the probability of two red balls being pulled out of bin A one after the other (without replacement)?

Solution:

Bin A contains 3 white and 2 red balls.

$$Pr(RR) = \frac{2 \cdot 1}{5 \cdot 4}$$

■

(b) We take out all 12 ball, then add another extra ball. What is the maximum number of bins that these 13 balls can be distributed into, such that at least one bin has at least 5 balls in it?

Solution:

We use the general form of the pigeonhole principle, this can be calculated by computing the largest value of k such that $\lceil \frac{13}{k} \rceil \geq 5$. For $k = 3$, this gives us $\lceil \frac{13}{3} \rceil = \lceil 4.33 \dots \rceil \geq 5$. Thus, 3 bins is the maximum to ensure that there's a bin with ≥ 5 balls in it.

We can also verify what happens if there are 4 bins. In that case, the best guarantee we can provide is $\lceil \frac{13}{4} \rceil = \lceil 3.75 \rceil = 4$ balls in at least one bin.

■

(c) What is the joint probability table of picking one ball from bin A and one from bin B? Recall: Bin A has 3 white and 2 red balls. Bin B has 4 white and 3 red balls.

Solution:

	$A = \text{White}$	$A = \text{Red}$
$B = \text{White}$	$\frac{3}{5} \cdot \frac{4}{7}$	$\frac{2}{5} \cdot \frac{4}{7}$
$B = \text{Red}$	$\frac{3}{5} \cdot \frac{3}{7}$	$\frac{2}{5} \cdot \frac{3}{7}$

Table 2: Joint probability table of picking a single ball each from A and B

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USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.