CS1800 Discrete Structures

Solutions to Exam: 1

Strange & Varma *Date: October 2, 2026*

- **Logistics:** Read all these instructions. Write your name on the top of every page. **Write your ID and Name in the blank spaces provided above the points table on this page.**
- **Duration, Problems and Points:** This exam lasts for **90 minutes** and has **7problems** totaling **90points**. Make sure to read all problems to decide an order to attempt them that would allow quickest progress.
- **Partial Credit:** Points are determined on the basis of both the correctness and the clarity of your solution. **Show your work**, as partial credit may be given for correct approach even without the final answer. Present the best solution/approach you can, even if it doesn't meet all desired properties.
- **Reference sheet:** You are allowed to bring **one US letter (8.5x11–inch)** cheat sheet, with anything written/typed on it; **only one side** of the sheet can be used. **No other materials or devices are permitted. No collaboration of any kind is permitted.**
- **Policy on cheating:** Students who violate the above rules on scholastic honesty are subject to disciplinary penalties. Any student caught cheating will *immediately* receive an **F** (failing grade).

  Sign below and confirm that you are strictly abiding by the rules of this test.

Signature: _____

ID: _____

NAME: _____

Question	1	2	3	4	5	6	7	Total
Points	20	10	10	10	10	20	10	90

Best of luck!

Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Annihiltor/Domination Laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotence Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Complement/Negation Laws	$p \wedge \neg p \equiv F$ $p \vee \neg p \equiv T$
Double Negation Law	$\neg\neg p \equiv p$
Absorption Laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
De Morgan's Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Table 1: Laws of Boolean Algebra for reference.

Problem 1 (Boolean Formula ≡ Logical Expression) 20

In this simplification, on each line, write the law being used in the proof of equivalence or fill in the missing step given the name of the law. *Note: It is not necessary to perform commutative and associative law application in separate steps; they do not need to be mentioned.*

Solution:

$$(C \wedge \neg A) \vee \neg A \vee \neg A \vee \neg\neg\neg B$$

$= \neg A \vee \neg A \vee \neg\neg\neg B$ Absorption

$= \neg A \vee \neg\neg\neg B$ Idempotent

$= \neg A \vee \neg B$ Double Negation

$= \neg(A \wedge B)$ DeMorgan's

■

Problem 2 (Truth Table $\longrightarrow$ Circuit) 10

Draw the circuit (using only AND, OR, and NOT gates) corresponding to the Truth Table in Table 2. A, B are inputs, the last column is the output and the rest are intermediate outputs of logic gates.

Table 2

A	B	X	Y	Output
0	0	1	0	0
0	1	0	1	0
1	0	1	1	1
1	1	0	1	0

Solution:

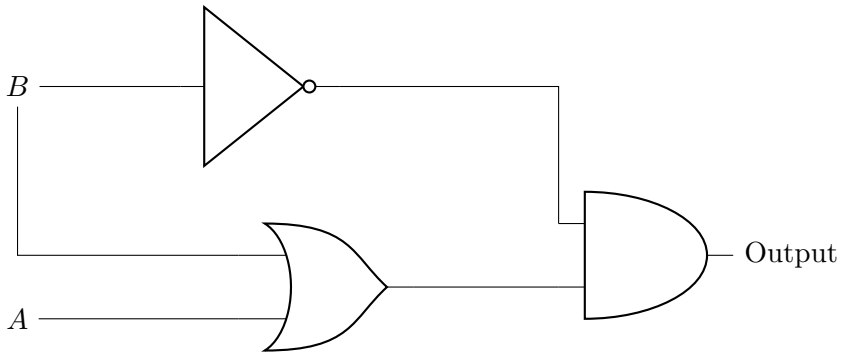


Figure 1: Circuit based on Truth Table in Table 2

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Problem 3 (Circuit $\longrightarrow$ Logical Expression) (6+2+2)

Convert the circuit in Figure 2 to a logical expression using the boolean variables a, b, and c.

Provide truth values for a, b, and c such that the output is True and another set of truth values for a, b, and c such that the output is False. If no truth values work, then say "None".

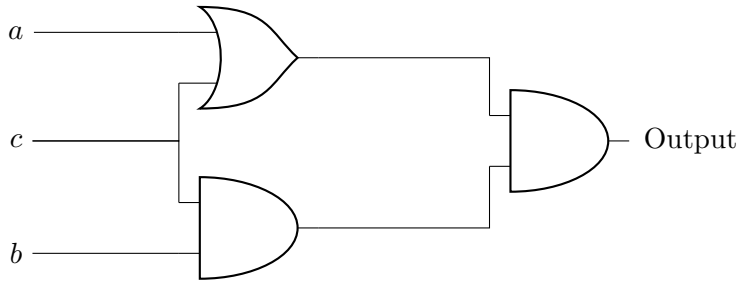


Figure 2

Solution:

$$(a \vee c) \wedge (b \wedge c)$$

For **True** output set $a = b = c = T$

For **False** output set $a = b = c = F$ ■

Problem 4 (Logical Expression $\longrightarrow$ Truth Table) 10

Are the statements 1 and 2 below logically equivalent? If they are, show their truth table; if they are not, give an example input on which they do not agree.

Statement 1: $(\neg a \vee b) \wedge (\neg a \vee \neg b)$ Statement 2: $(\neg a \wedge \neg b) \vee (\neg a \wedge b)$

Solution:

a	b	$\neg a$	$\neg a \vee b$	$\neg a$	$\neg b$	$\neg a \vee \neg b$	$(\neg a \vee b) \wedge (\neg a \vee \neg b)$
0	0	1	1	1	1	1	**1**
0	1	1	1	1	0	1	**1**
1	0	0	0	0	1	1	**0**
1	1	0	1	0	0	0	**0**

Table 3: Statement 1's Truth Table

a	b	$\neg a$	$\neg b$	$\neg a \wedge \neg b$	$\neg a$	$\neg a \wedge b$	$(\neg a \wedge \neg b) \vee (\neg a \wedge b)$
0	0	1	1	1	1	0	**1**
0	1	1	0	0	1	1	**1**
1	0	0	1	0	0	0	**0**
1	1	0	0	0	0	0	**0**

Table 4: Statement 2's Truth Table

The output columns (last columns) of the two Truth Tables (Table 3 and Table 4) match, hence the two statements are equivalent.

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Problem 5 (Implications) (6 + 4)

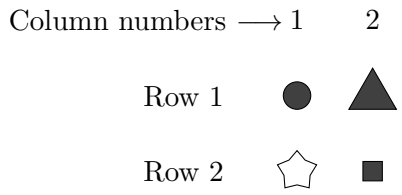


Figure 3: Universe

For the following statement, provide its contrapositive, converse, and inverse (all in English, no logical operators required). For each of the 4 statements, determine its truth value based on Figure 3.

Solution:

Original Statement: "If the shape is a square then it is not clear."

Truth Value: *True*

Contrapositive: "If the shape is clear then it is not a square."

Truth Value: *True*

Converse: "If the shape is not clear then it is a square."

Truth Value: *False*

Inverse: "If the shape is not a square then it is clear."

Truth Value: *False* ■

Problem 6 (Predicates and Quantifiers) (8+2) x2

In the following, you can use the predicates $Square(x)$, $Circle(x)$, $Star(x)$ to denote whether a given shape x is a square, circle or star respectively. Similarly, $Shaded(x)$ and $Clear(x)$ are predicates denoting that a shape has been shaded or is clear (unshaded). $Row(x)$ is the row number (from 1, 2, 3) of a given shape x. $Col(x)$ is the column number (from 1, 2, 3, 4) of a given shape x.

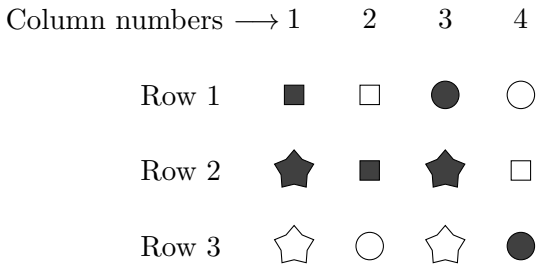


Figure 4: Universe

For the following statements, if they are in English, translate to a boolean formula and vice versa. For each statement, also mention if it is True/False based on Figure 4. Provide a counterexample (using row/column numbers to refer to specific shapes) if the statement is False.

(a) "Every circle is clear."

Solution:

$$\forall x : Circle(x) \implies Clear(x)$$

False, there's a circle in row 1 column 3 which is shaded. ■

(b) $\forall x, \exists y : Shaded(x) \wedge Circle(x) \implies Clear(y) \wedge Circle(y) \wedge \Big(Row(x) = Row(y)\Big)$

Solution:

"For every shaded circle, there is a clear circle in the same row."

True. Rows 1 and 3 are the only columns with shaded circles, they also contain clear circles. ■

Problem 7 (`bin2dec` and `dec2bin`) 5+5

(a) Convert 11010_2 to decimal; show all steps.

Solution:

$$\begin{aligned} 11010_2 &= 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \\ &= 1 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 0 \cdot 1 \\ &= 16 + 8 + 2 \\ &= 26_{10} \end{aligned}$$

■

(b) Convert 28_{10} to base 2 using the division method; show all steps.

Solution:

$$\begin{aligned} 28_{10} &\longrightarrow (14, 0) \\ &\longrightarrow (7, 0) \\ &\longrightarrow (3, 1) \\ &\longrightarrow (1, 1) \\ &\longrightarrow (0, 1) \end{aligned}$$

$\implies 28_{10} = 11100_2$

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USE THIS PAGE FOR SCRATCH WORK.
IF CONTINUING AN ANSWER, CLEARLY INDICATE THE QUESTION NUMBER.