Professor Hamlin
Day 6

Agenda

1) Review
2) Sets
   - vocab
   - builder notation
   - set operations → union, intersection, complement
     difference

Review

Extended Conditionals

original = contrapositive, inverse = converse (

Double implication

negate condition $\neg(x \to y) = x \wedge \neg y$

Extended Quantifiers

Negating quantifiers

Nested quantifier

Exercises:

1) Given the following conditional identify these variants: "If I am sleepy then I nap" (x = sleepy, y = nap)

a) If I'm not sleepy then I don't nap.

$\neg x \to \neg y$, inverse

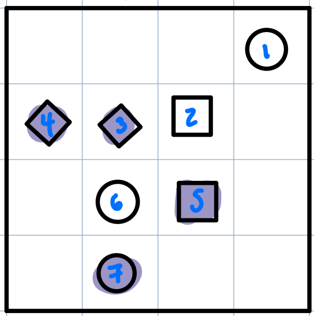
b) If I don't nap then I'm not sleepy

$\neg y \to \neg x$ contrapositive

c) If I nap then I am sleepy and if I'm sleepy I nap

$\underbrace{y \to x}_{\text{converse}} \wedge x \to y$ double implication

2) Tarski World



State if the following are true/false
if false provide counterexample

a) $\forall x \exists y$: circle(x) ∧ square(y) ∧ x Rightof y (x,y)

F, 6, 7 don't have sq right of them

$\forall x \exists y$
x choose y that works

$\exists x \forall y$
same x for all y

b) $\exists x \forall y$: circle(x) ∧ square(y) ∧ x is Above y (x,y)

True

Sets

So far what x/y/z etc have been has been confusing

$\forall x$: Yellow(x)

is x a pet, a student an object?

Need a way of defining what x is → as well as talking about groups in general

Sets

Set: a unordered collection or group of unique items

$\{1, 2, 3\}$ or $\{a, b, c\} = \{b, a, c\}$

$\{1, 2, 3, a, b, c\}$

Notation note: $\{1, 2, 3, 4\} = \{1, 2, 3, 4, 4, 4\}$
these two are equivalent but it is bad form to have an object show up more than once

Common sets:

- $\emptyset = \{\}$ empty set w/ no items

$\mathbb{Z}$

- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2 \ldots\}$ integers

sets can be infinitely large

$\mathbb{N}$

- $\mathbb{N} = \{0, 1, 2, 3 \ldots\}$ natural numbers

sometimes excluded :/

- $\mathbb{R} = \{-2, 0, 1\frac{1}{2}, \pi \ldots\}$ Real numbers

Set membership

An object is either in or out of a set

used to denote set membership

$x \in \mathbb{Z}$ x in the integers

$x \notin \mathbb{N}$ x not in the natural #s

These are both boolean statements - either true or false

Exercise True or false

1) $0 \in \mathbb{R}$ T
2) $-1 \notin \mathbb{N}$ T
3) $0 \in \emptyset$ F
4) $2.5 \in \mathbb{Z}$ F
5) $5 \in \mathbb{N}$ T
6) $-\pi \notin \mathbb{R}$ F

Set builder notation

Listing out items, or if there is no easy pattern like the reals, is hard. Solution? Set builder notation.

$$A = \{ x \in \mathbb{N} \mid (3 \le x) \wedge (x \le 5) \}$$

s.t.

A is the set of x in the natural numbers such that x is greater or equal to three and less than or equal to five

The construction is

$$A = \{ x \in \text{Some larger Universe} \mid \text{Some predicate about } x \}$$

if the predicate is T, x is in the set A

F, x is not in the set A

$$\{ x \in \text{CS1800Students} \mid \text{Cool}(x) \}$$

The set of all students x in CS1800 who are cool

Exercise

1) List all items in this set

$$A = \{ x \in \mathbb{Z} \mid |x| < 5 \}$$ absolute value

$\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

2) Write w/ set builder notation

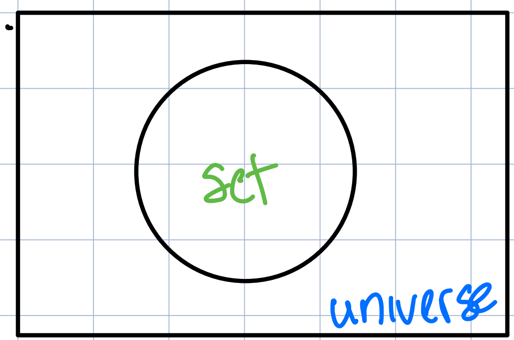
B = set of natural # x, s.t. x mod 3 = 0 and x mod 7 = 0 and x < 40

$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \wedge (x \bmod 7 = 0) \wedge (x < 40) \}$

3) $2 \in \{ y \in \mathbb{Z} \mid y > 4 \}$?

Venn diagrams:

A way of visually representing sets



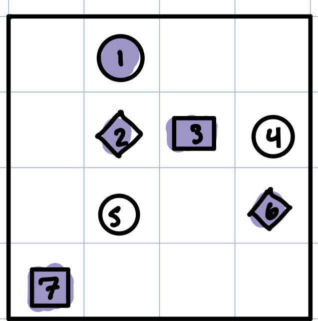
universe: larger set e.g. all things in the universe, students at NEU

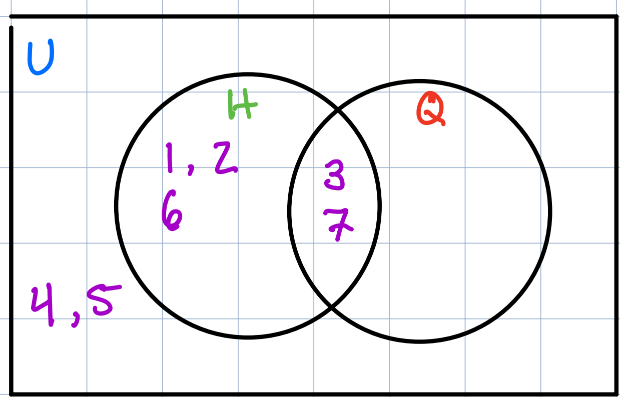
Example

U = all shapers

H = shaded shapes

Q = squares





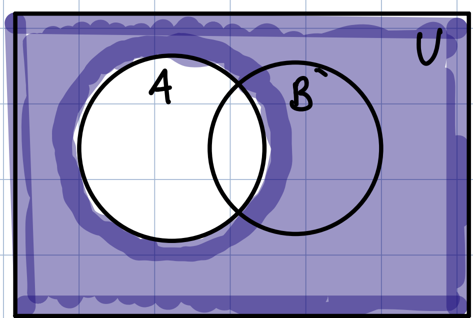
Note: just because an area exists doesn't mean it has items in it!

Set operations:

Logic Operators	NOT $\neg$	AND $\wedge$	OR $\vee$	XOR
Set operators	Complement $\sim, c$	intersection $\cap$	union $\cup$	sym diff

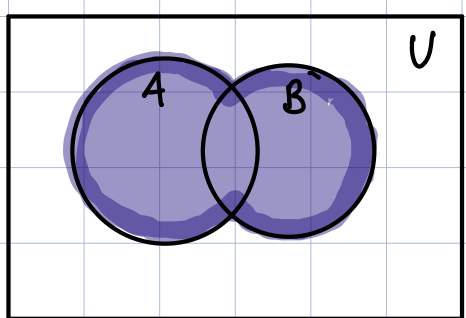
Complement : $\bar{A} = A^c = \{ x \in U \mid x \notin A \}$

All x in universe s.t. x is not in A



Union : $A \cup B = \{ x \in U \mid x \in A \vee x \in B \}$

All x in universe s.t x in A <u>or</u> x in B.



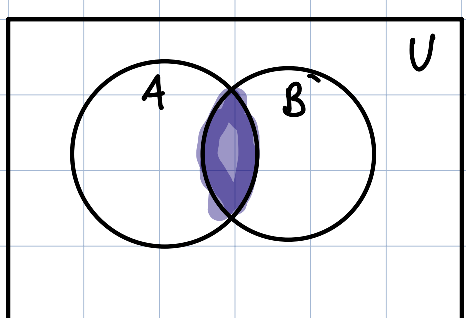
$\cup$ – U for union

Rather like OR

$A \cup B = B \cup A$

Intersection: $A \cap B = \{ x \in U \mid x \in A \wedge x \in B \}$

A x in universe s.t. x in A and x in B



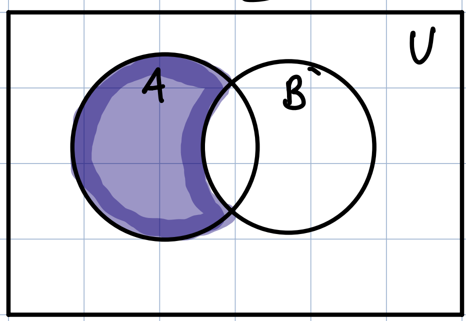
Rather like and

$A \cap B = B \cap A$

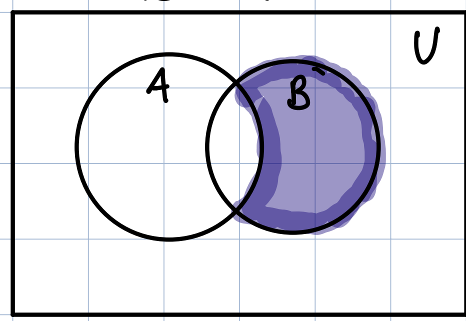
Difference: $A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$

all items in one set (A) not the other (B); order matters!

A - B

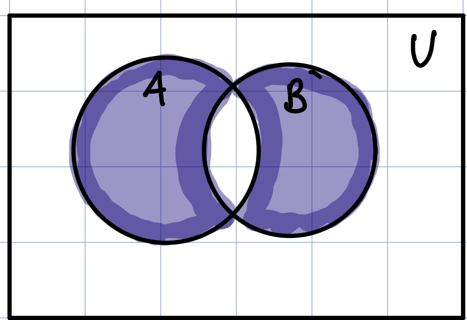


B - A



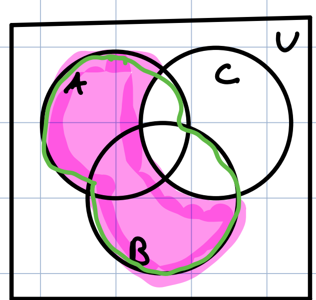
Symmetric Difference: $A \Delta B = \{x \in U \mid x \in \{A \cup B\} \wedge x \notin (A \cap B)\}$

all items in one set or other, not both (XOR)



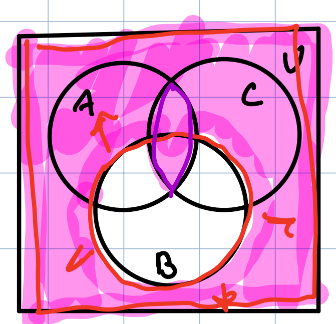
$x \in A$	$x \in B$	$x \in A \Delta B$
F	F	F
F	T	T
T	F	T
T	T	F

Exercise: Shade areas for Venn diagram

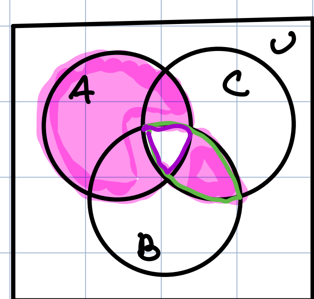


$(A \cup B) \overset{\text{diff.}}{-} C$

Order of op
PNAXO
P·comp·int·sym diff·union



$(A \cap C) \cup \overline{B}$



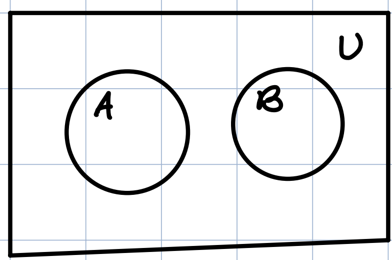
$A \Delta (B \cap C)$

Set Relationships

$x > y$, $x \geq y$, $x \neq y$, $x = y$, $x < y$, $x \lneq y$

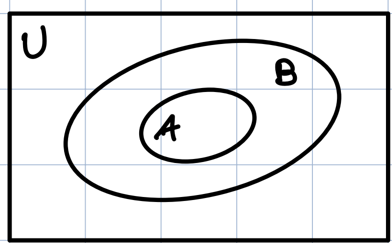
Disjoint set: A & B are disjoint if no item is in both sets. OR $A \cap B = \{\} = \emptyset$

$x \neq y$



Subset: A is subset of B if All items in A are also in B

$\boxed{x \leq y}$



$A \subseteq B = x \in A \rightarrow x \in B$

if x in A then x in B

$A \subseteq A$, a set is a subset of itself

Set Equality: $A = B$ if A is subset of B and B is subset of A

$\boxed{x = y}$

$x \leq y$
$y \leq x$
$\rightarrow x = y$

A & B have same items.

$A \subseteq B \wedge B \subseteq A \Rightarrow A = B$

Strict (Proper) Subset

$\boxed{x < y}$

$x \leq 3$ includes 3

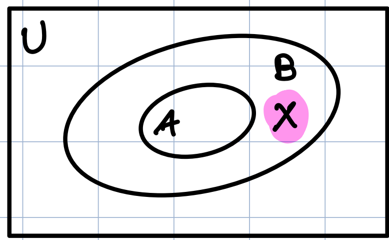
$x < 3$ does not include 3

same for sets → want to say set is strictly smaller

$A \subseteq B$ A might equal B

$A \subset B$ B has items A does not

$A \subset B$ = All items in A also in B AND B contains an item not in A $\quad A \subseteq B \wedge B - A \neq \emptyset$



$\in$ vs $\subseteq / \subset$

$$\boxed{\text{Note: only use } \subseteq, \subset \text{ with sets on both sides} \quad \text{Not } \in}$$

Exercise: $A = \{1, 3, 5, 7\}$, $B = \{x \in \mathbb{Z} \mid 1 \leq x < 10\}$
$C = \{2, 4, 6, 8\}$

1) $5 \in \underbrace{A \cap B}_{\{1,3,5,7\}}$ T
2) $A \subset B$ T
3) $B \subseteq B$ T
4) $C \in B$ wrong notation (F)
5) $B \subset \underbrace{A \cup C}_{\{1-8\}}$ F, B has 9
6) $C \subset \underbrace{B - A}_{\{2,4,6,8,9\}}$, T

$$\boxed{\text{Note } \emptyset \subseteq A \text{ is always true}}$$

Other Terminology

Cardinality: number of unique items in set

$|A|$ = cardinality of set A $\qquad a = \emptyset$, $\{a\}$

Ex) $A = \{1, 2, 3, 4\}$ $\quad B = \emptyset$ $\quad C = \{\emptyset\}$
$|A| = 4$ $\quad |B| = 0$ $\quad |C| = 1$

Sets can contain items that are other sets!

$x = \{1, 2\}$

$D = \{\{1,2\}, 3, 4\}$ $\quad$ $E = \{\{1,2,3,4\}\}$ $\quad$ $F = \{\mathbb{R}, \mathbb{Z}\}$

$|D| = 3$ $\quad$ $|E| = 1$ $\quad$ $|F| = 2$

$D = \{x, 3, 4\}$

$G = \{\{1\}, \{1\}\}$ $\quad$ $|G| = 1$

Powerset: The set of sets that can be made with original set

$A = \{1, 2\}$ $\qquad$ $\mathbb{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

Another way to think about it...

$\mathbb{P}(A) =$

1	2	
F	F	$\{\emptyset,$
F	T	$\{2\},$
T	F	$\{1\},$
T	T	$\{1,2\}\}$

Exercise: $A = \{1, 5, 7\}$

1) $|A| = 3$ $\qquad$ 2) $|\mathbb{P}(A)| = 8$

1	5	7	
F	F	F	$\emptyset$
F	F	T	$\{7\}$
	⋮		$\{5\}$
	⋮		$\{5,7\}$
	⋮		$\{1\}$
	⋮		$\{1,7\}$
	⋮		$\{1,5\}$
T	T	T	$\{1,5,7\}$