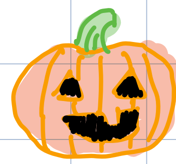
Professor Hamlin
Day 15

Agenda

1. Admin - HW5 probability due Friday
2. Review
3. Graph Definitions
4. Graph representation
   - list of lists
   - adjacency matrix
5. Graph equivalence (isomorphism)



Happy Halloween

Review

Bernoulli: outcome of experiment w/ success / failure

Binomial: odds of succeeding k successes out of N trials

Assumpt.: 1) each trial independent
2) same p for each trial

Poisson: odds of k events over some window (λ)

Assumpt. 1) rate constant
2) events are independent

Exercise

1. "Probability of drawing 5 red cards from 52 card deck"

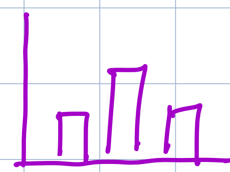
Which distribution & does it satisfy assumptions?

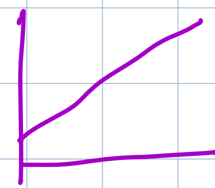
Binomial? rate constant? not

2. "Odds of 4 students coming to my office hours?"

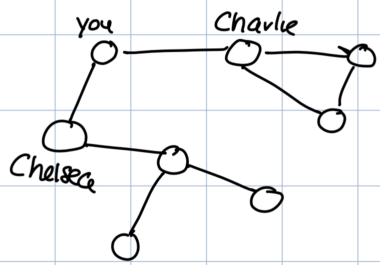
Poisson, yes seems safe

Graphs :



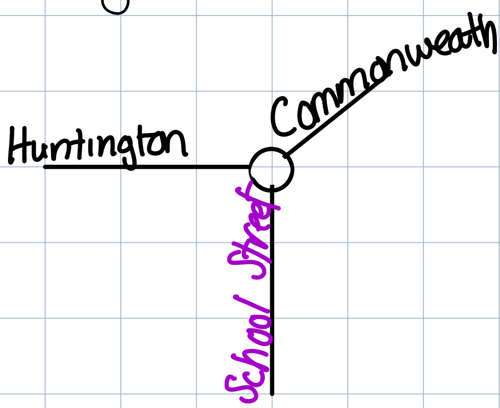


Real life examples:



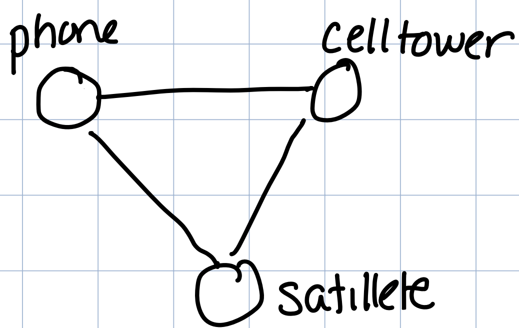
Social networks

people connected by friendship



Streets

intersections connected by roads



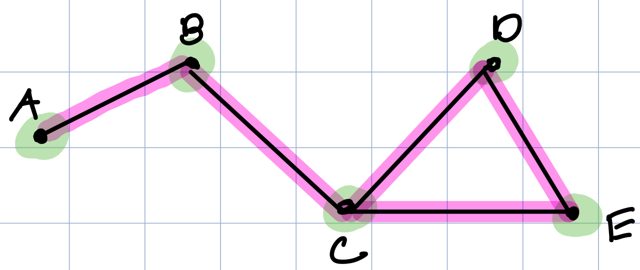
Internet

routers & devices connected by communication

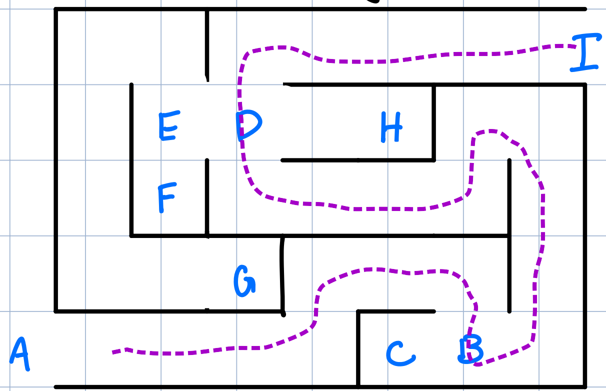
> Graphs are a fundamental way of representing real data & structures

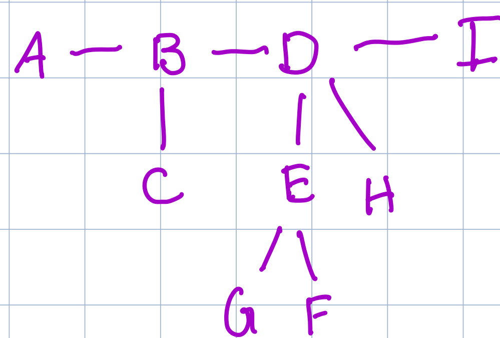
What are graphs (the data structure)

A set of vertices (nodes) and a set of edges connecting pairs of vertices



Example Node - intersection, deadend
Edge - possible movement





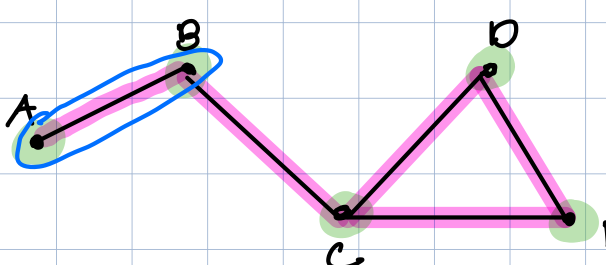
> Clear representation helps us solve the problem

Formally representing graphs $G = (V, E)$

ordered set of things

set of vertex

set of edges



Vertices

$V = \{A, B, C, D, E\}$

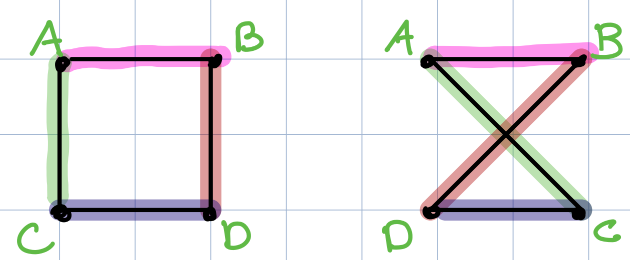
Edges

$E = \{\boxed{(A, B)}, (B, C), (C, D), (C, E), (D, E)\}$

V = set of vertex names

E = set of tuples (pairs) of vertex names representing edges

Graph can be drawn differently and still be the same graph



$G = (V, E)$

$V = \{A, B, C, D\}$

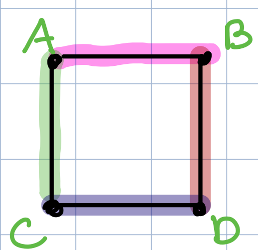
$E = \{(A,B), (A,C), (B,D), (C,D)\}$

Warning: lots of vocab, most is intuitive but check just to make sure

Definitions

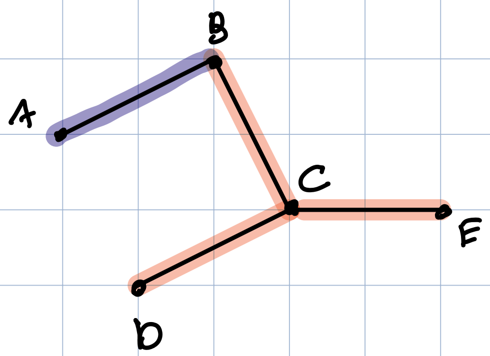
Adjacent: two vertices are adjacent in a graph if they are connected by an single edge

Adjacent: A & B   Not: A & D
A & C   C & B



- a vertex and edge are adjacent if vertex is in edge
  Adjacent: A & (A,B)   Not D & (A,C)
- two edges are adjacent if they share a vertex
  adjacent: (A,B) & (B,D)   Not: (A,B)
  (C,D)

Degree of vertex: number of edges which are adjacent to it.



Deg(A) = 1

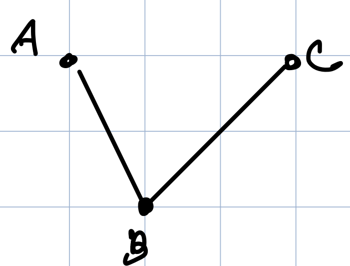
Deg(C) = 3

Deg(B) = 2

total degree: sum of the degrees of all the vertices

Total Deg = Deg(A) + Deg(B) + Deg(C) + Deg(D) + Deg(E) = 8
(1, 2, 3, 1, 1)

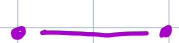
Exercise | Draw a graph were the total degree is odd (or argue why it isn't possible)



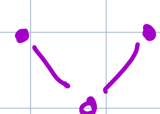
Deg(A) + Deg(B) + Deg(C)
1 + 2 + 1 = 4

What is the relationship between the total degree and the number of edges in the graph? (try drawing some examples)

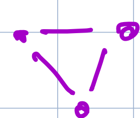
0 E
1 V
0 TD



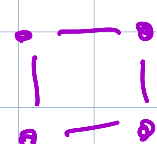
1 E
2 V
2 TD



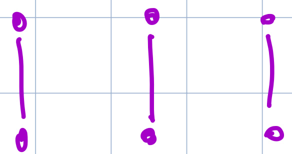
2 E
3 V
4 TD



3 E
3 V
6 TD



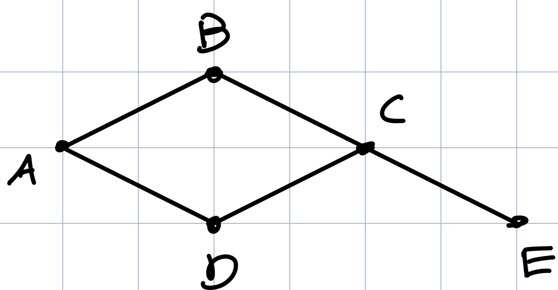
4 E
4 V
8 TD



3 E
6 V
6 TD

Total Degree = 2 • (Total # Edges

-



Walk: a sequence (ordered list) of adjacent edges (or equivalently adjacent vertices)

(A, B) (B, C) (C, B) = A B C B

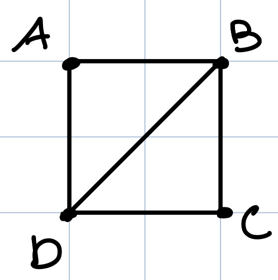
Path: a walk where each vertex is unique

Path: A B C E   Not: A B C B

Cycle: a path which starts & ends at same vertex (only one allowed to be not-unique)

Cycle: A B C D A   Not: A B C E

Exercise)



identify the following as: cycle, path, or walk

1. A B D C

Path

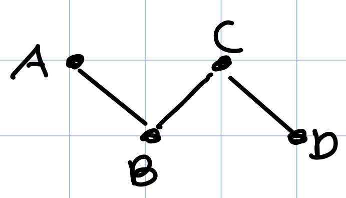
2. A D C B D

Walk

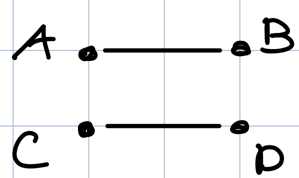
3. A B D A

Cycle

Connected Graph: a graph is connected if there is a path from every node to every other node

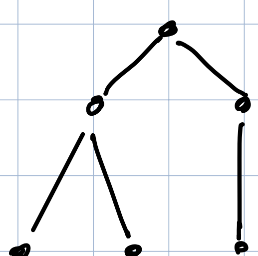


Connected

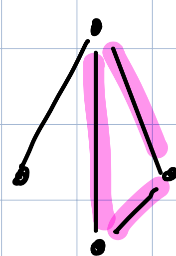


Not connected

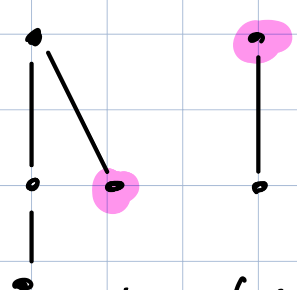
Tree: a connected graph without any cycles (acyclic)



Tree



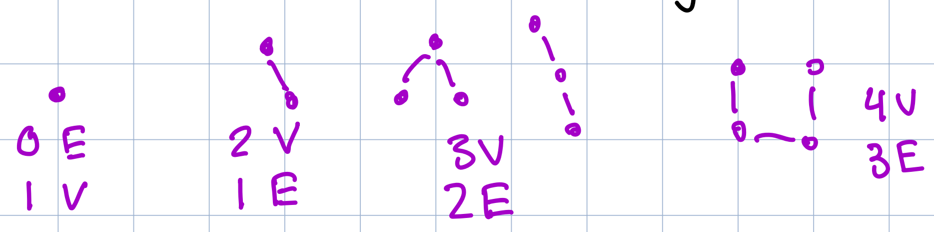
Not (has cycle)



Not (disconnected)

Trees are super common kinds of graphs

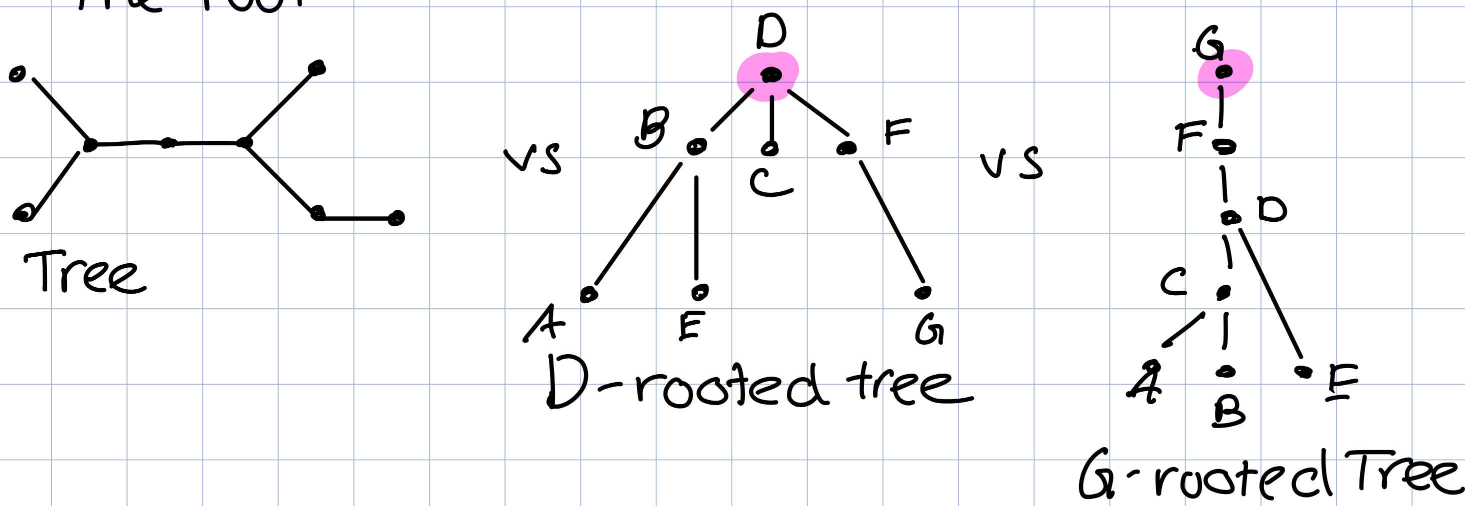
<u>Exercise</u> What is the relationship between the $|V|$ (number of vertices) and $|E|$ (number of edges)?



For trees:

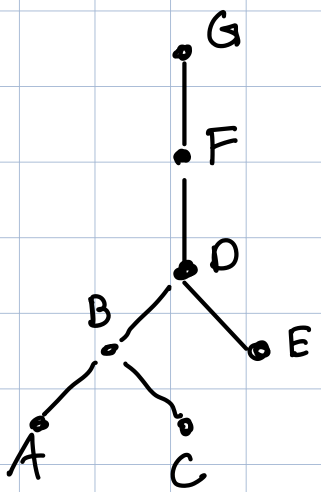
$$|E| = |V| - 1$$

Rooted Trees: a tree (connected, acyclic graph) which has a specific vertex identified by the root



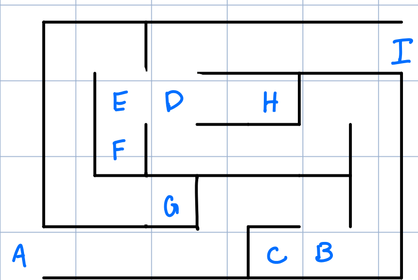
Convention: root of tree is drawn on top

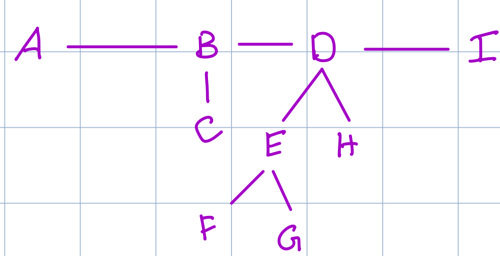
Why care about rooted trees? Allows us to define relationships between vertices



- **Parent** (of x): next vertex on path to the root (when has no parent)
  e.g. D is parent of B
- **Children** (of x): set of all vertices that x is a parent of
  e.g. {B, E} are D's children
- **Leaf**: a vertex with no children
  e.g. A, C, E are leaves
- **Sibling** (of x): set of vertices that have same parent as x
  e.g. C is A's sibling
- **Ancestor** (of x): all vertices along path from x to the root
  e.g. B, D, F, G are A's ancestors
- **Descendent** (of x): all nodes that have x as an ancestor
  e.g. B, E, A, C are D's descendents

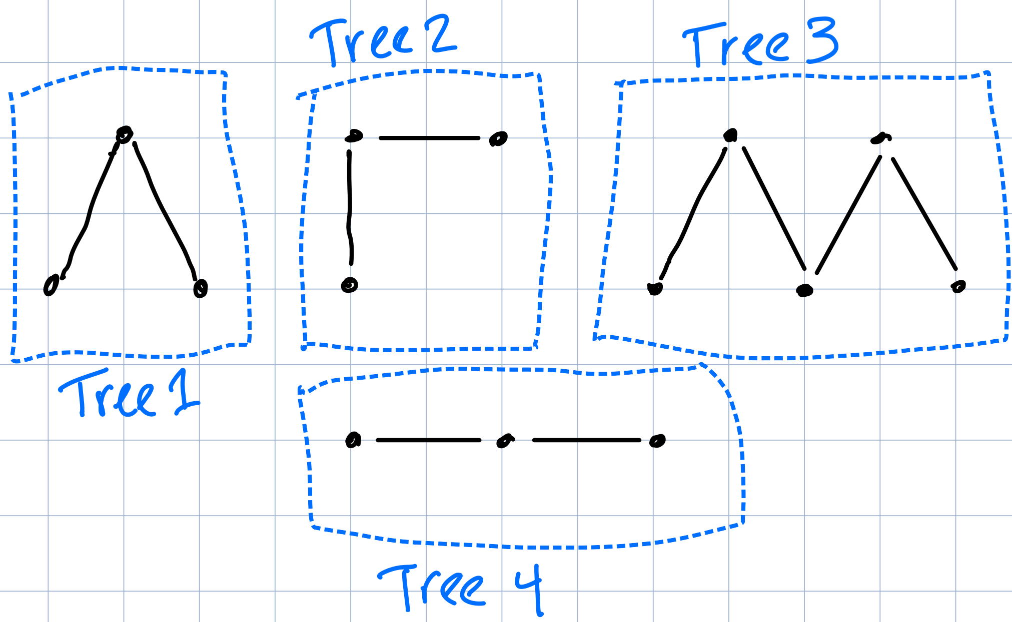
Our maze was a rooted tree w/ A as a root!



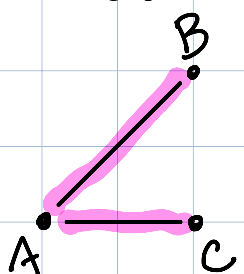


We can also have a graph made of several trees

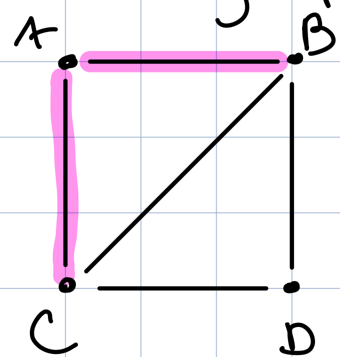
Forest: any acyclic graph (notice doesn't have to be connected!)



Subgraph: a graph whose vertices & edges are contained within another graph



is subgraph



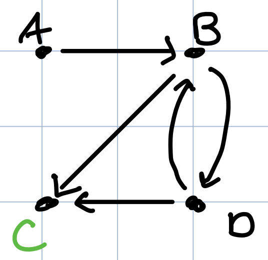
$G1 = (V_1, E_1)$

$G2 = (V_2, E_2)$

$$\boxed{\begin{array}{c} V_1 \subseteq V_2 \\ E_1 \subseteq E_2 \end{array}}$$

We so far have been defining general graphs but there are a few special cases

Directed: each edge has direction (e.g. one way roads)

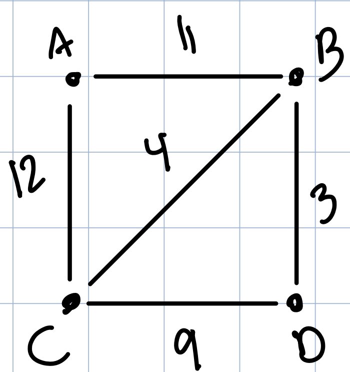


$V = \{A, B, C, D\}$

$E = \{(A,B), (B,C), (D,C)$
$(B,D), (D,B)\}$

order indicates directi

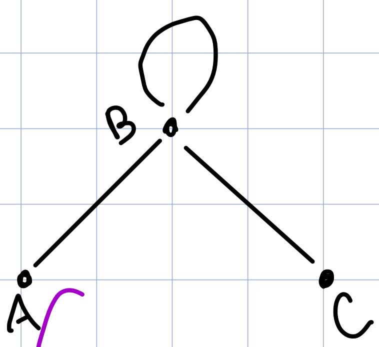
Weighted: each edge has a weight



$V = \{A, B, C, D\}$

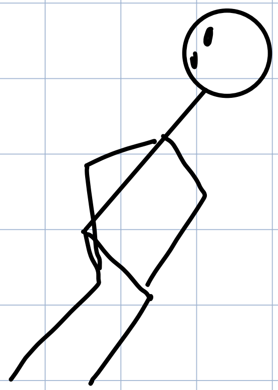
$E = \{(A,B,11), (A,C,12), (C,B,4)$
$(C,D,9), (B,D,3)\}$

Non-simple: edge may start/end at same vertex



$V = \{A, B, C\}$

$E = \{(A,B), (B,B), (B,C)\}$

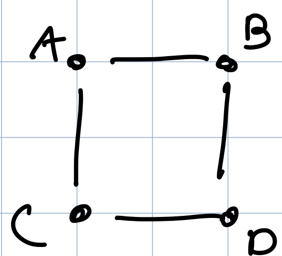


Let's take a breath, a lot of vocab

Good news: almost done
can just look back if you have questions

Bad news: sometimes def'ns are inconsistent (vertex vs. node, is a vertex it's own ancestor)

Neighbors: two vertices are neighbors if they are adjacent (connected by an edge)



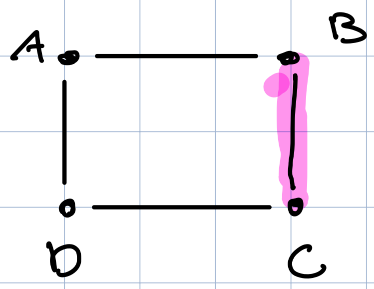
A & B Neighbors

A & D Not

How do we represent graphs on computers? (remember computers think in 0/1)

Approach 1: Adjacency list

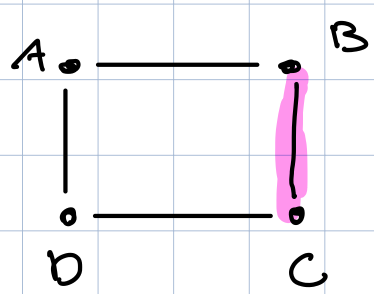
idea: just list neighbors for each vertex



A: [B, D]
B: [A, C]
D: [A, C]
C: [B, D]

Approach 2: Adjacency Matrix

idea: have matrix, $|V| \times |V|$, 1 in row i, column j means edge between i & j



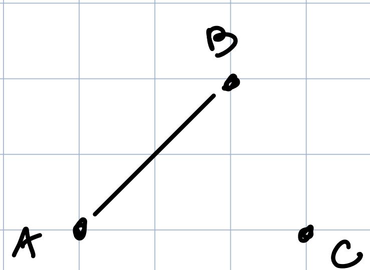
	A	B	C	D
A	0	1	0	1
B	1	0	1	0
C	0	1	0	1
D	1	0	1	0

There is edge between B,C

Convention: a node is not it's own neighbor

Notice the symmetry

Exercise Given one representation of a graph construct the other two (image, list, matrix)

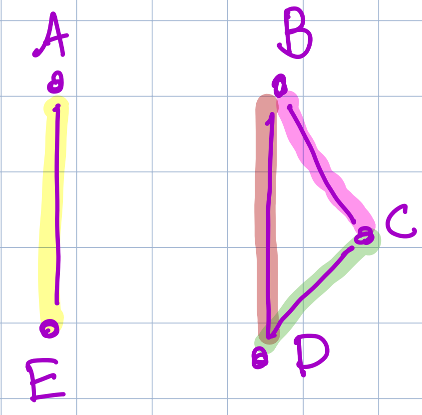


A: [B]
B: [A]
C: [ ]

	A	B	C
A	0	1	0
B	1	0	0
C	0	0	0

	A	B	C	D	E
A	0	0	0	0	1
B	0	0	1	1	0
C	0	1	0	1	0
D	0	1	1	0	0
E	1	0	0	0	0

A: [E]
B: [C, D]
C: [B, D]
D: [B, C]
E: [A]

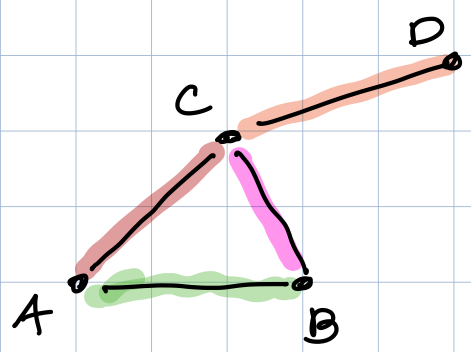


Graph isomorphism:

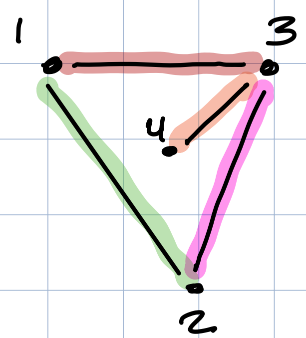
iso morphic
"same" "shape"

<u>Idea</u>: two graphs are isomorphic when they have the same shape

e.g. when we can rename* the nodes of one to get another



A = 1
B = 2
C = 3
D = 4



* rename = one-to-one mapping (bijection)