Professor Hamlin
Day 7

Agenda

1) Review
2) Computer representation of sets
3) Set and Logic Algebra
4) Digital circuits

Review

Sets: unique unordered

set builder notation, venn diagrams

set operation $\cup, \cap, {}^c, -, \Delta$

cardinality & powersets

Exercise:

1) $A = \{\boxed{\{1,2\}}, 3, 4\}$ $\quad B = \{1, 2, \boxed{3}\}$

a) what is $A \cup B$?

$\{\{1,2\}, 3, 4, 1, 2\}$

b) what is $|A \cap B|$? $A \cap B = \{3\}$

1

Bonus c) $\{1,2\} \in A - B$? $\{\{1,2\}, 4\}$

Yes

Bonus d) $\{1,2\} \subseteq A \Delta B$? $\{\{1,2\}, 4, 1, 2\}$ $\{1,2\}$

Yes

Representing sets on computers

Remember how we figured out power sets? $A = \{a, b\}$ $\mathcal{P}(A) = \{\{a\}, \{b\}, \{a, b\}, \emptyset\}$

$\{a,b\} = \{b,a\}$

a	b	$\mathcal{P}(\{a,b\})$
F	F	$\emptyset$
F	T	$\{b\}$
T	F	$\{a\}$
T	T	$\{a, b\}$

Computers like things that are binary and fit in a set amount of bits

How can we do this for sets?

1 a	2 b	$\mathcal{P}(A)$	
F → 0	F → 0	$\emptyset$	→ 0 0
F → 0	T → 1	$\{b\}$	→ 0 1
T → 1	F → 0	$\{a\}$	→ 1 0
T → 1	T → 1	$\{a, b\}$	→ 1 1

step 1 → assign index position to all elements in universe

step 2 → if item 1 in set → index i set to 1

not in set → index i set to 0

e.g. $U = \{$green, black, red, white, blue$\}$
1 2 3 4 5

$A = \{$ black, red, blue$\}$
0 1 1 0 1

computer store $A = 01101$ $B = 10011$
# of bits = |Universe|

Connecting set operators to logic op

$U = \{$green, black, red, white, blue$\}$
1 2 3 4 5

$A = \{$ black, red, blue$\}$
$a =$ 0 1 1 0 1

$B = \{$green, black, blue$\}$
$b =$ 1 1 0 0 1

$A \cup B = \{$green, black, red, blue$\}$
1 1 1 0 1

01101
OR 11001
11101

Sets	Logic
$A^c = \{$green, white$\}$ $\overline{A}$	$a = 01101$ $\neg a = 10010$ each bit negated
$A \cup B$	$a = 01101$ $b = 11001$ $a \vee b = 11101$ apply or to each bit

$A \cap B = \{black, blue\}$	$a = 01101$ $b = 11001$ $a \vee b = 01001$ apply <u>and</u> to each bit
$A \Delta B = \{green, red\}$	$a = 01101$ $b = 11001$ $a \text{ xor } b = 10100$ apply <u>xor</u> to each bit

Connecting set/logic operators

Set	Logic
^{c} (complement)	$\neg$ (negation)
$\cap$ (intersection)	$\wedge$ (and)
$\cup$ (union)	$\vee$ (or)
Δ (sym. diff.)	XOR

Set Algebra (Logic Algebra)

algebra ....

$$x(x+10) - 2x + 15 \quad \text{(distributive law)}$$
$$x^2 + 10x - 2x + 15$$
$$x^2 + 8x + 15$$

manipulating expressions to simplify them.
Can do same for Logic / Set expressions!
We have a few rules to help us.

$(9+1)+5 =$
$9+(1+5)$

Algebra

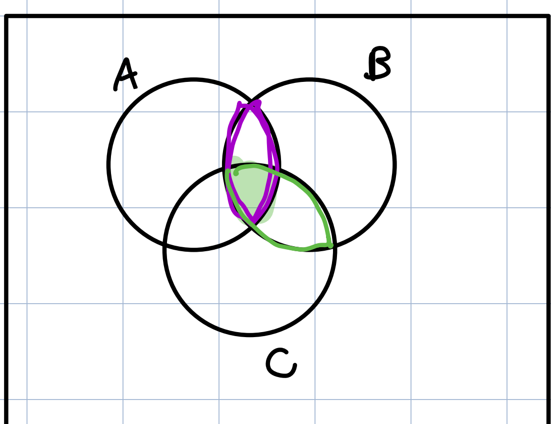
$(x+y)+z =$
$x+(y+z)$

p	q	r	$(p \vee q) \vee r$	$p \vee (q \vee r)$
F	F	F	F	F
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T

Logic

Associative Laws

(P ∨ Q) ∨ R = P ∨ (Q ∨ R)
(P ∧ Q) ∧ R = P ∧ (Q ∧ R)



Sets

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$\underbrace{(A \cap B)} \cap C = A \cap \underbrace{(B \cap C)}$$

Other Rules

Double Negation

¬ ¬ P = P

$$(A^C)^C = A$$

T → F → T

Distributive Laws

P ∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R)
P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

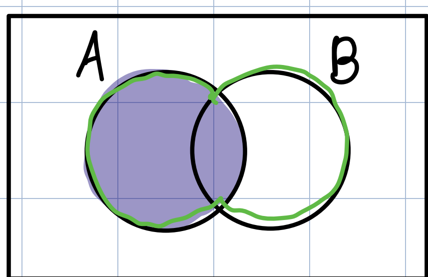
Idea: $x * (y + z) = x * y + x * z$

Absorption Laws

P ∧ (P ∨ Q) = P
P ∨ (P ∧ Q) = P

$$A \cap (A \cup B) = A$$
$$A \cup (A \cap B) = A$$

Idea:

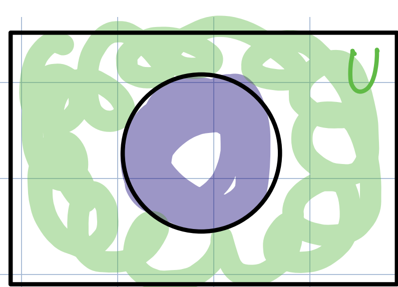


Complement Laws

P ∨ ¬ P = $\boxed{T}$
P ∧ ¬ P = $\boxed{F}$

$$A \cup A^C = U$$ universe
$$A \cap A^C = \emptyset$$ $\emptyset = \{\}$

Idea:



Idempotent Laws (you have overcomplicated things)

P ∨ P = P

P ∧ P = P

$A \cup A = A$

$A \cap A = A$

Identity

False ∨ P = P

True ∧ P = P

$\emptyset \cup A = A$

$U \cap A = A$

idea:

P	P∨F	P∧T	P∧F	P∨T
F	F	F	F	T
T	T	T	F	T

Domination:

True ∨ P = True

False ∧ P = False

$U \cup A = U$

$\emptyset \cap A = \emptyset$

p doesnot matter

DeMorgan's Laws

¬(P ∨ Q) = ¬ P ∧ ¬ Q

¬(P ∧ Q) = ¬ P ∨ ¬ Q

$(A \cup B)^C = A^C \cap B^C$

$(A \cap B)^C = A^C \cup B^C$

Recall from Day 4

Exercise Build Truth tables for

1) $\neg(A \vee B)$ 2) $\neg A \wedge \neg B$

A	B	A∨B	¬(A∨B)
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

A	B	¬A	¬B	¬A∧¬B
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

two statements are logically equivalent

Simplifying Boolean or set expressions

Ex) $(X \cup Y) \cap (X \cup \bar{Y})$

$xy + xz$
$x(y+z)$

(distributive law)

$X \cup (Y \cap \bar{Y})$

(complement law)

$X \cup \emptyset$

(identity law)

$\boxed{X}$

Ex) $\neg(\neg A \vee B) \wedge \neg B$

$(\neg\neg A \wedge \neg B) \wedge \neg B$ Demorgan

$(A \wedge \neg B) \wedge \neg B$ (double negation)

$A \wedge (\neg B \wedge \neg B)$ (associative)

$(P \wedge P)$

$A \wedge \neg B$ (idempotent

Practice 1) $(A \cup B) \cap \bar{A}$

$(A \cap \bar{A}) \cup (B \cap \bar{A})$ dist.

$\emptyset \cup (B \cap \bar{A})$ compl.

$B \cap \bar{A}$ identity

2) $(\neg x \wedge x) \vee (y \vee \neg\neg x)$

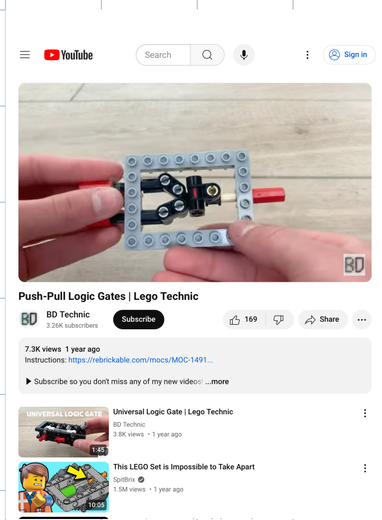
$F \vee (y \vee \neg\neg x)$ complement

$F \vee (y \vee x)$ double neg

$y \vee x$ identity

Circuits on Computers

https://youtu.be/RA2po1xk_0A?t=5

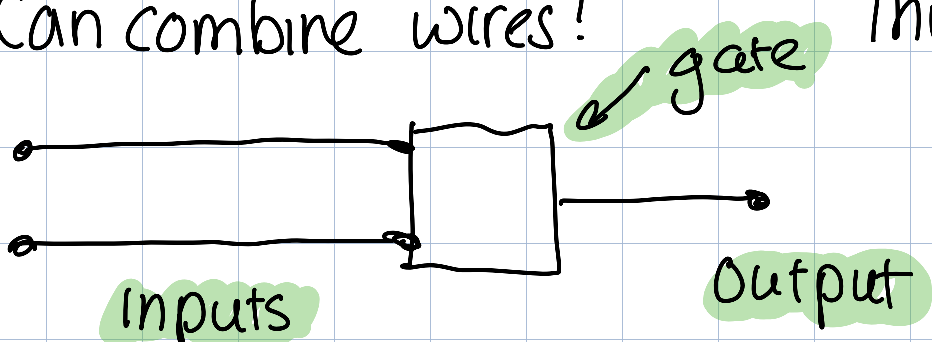


wire

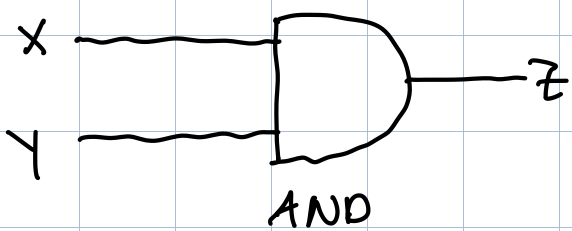
electricity = 1

no electricity = 0

Can combine wires! This forms a.

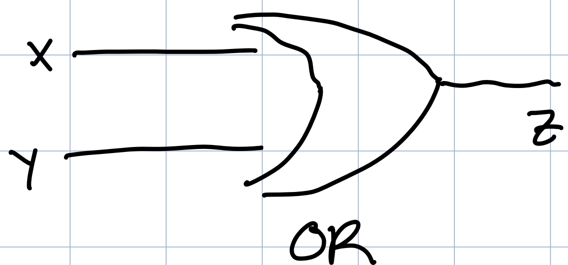


Each of our boolean operators has a corresponding gate

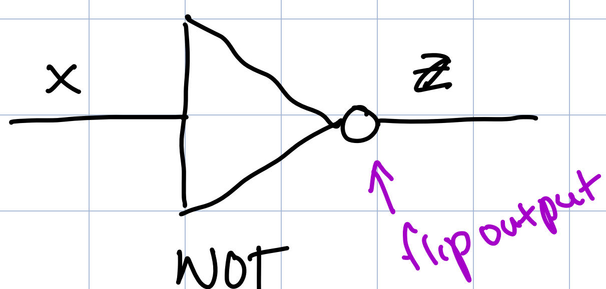


X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

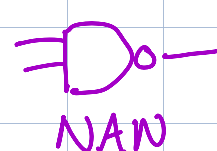
Note when talking about circuits we use 0/1 instead of T/F

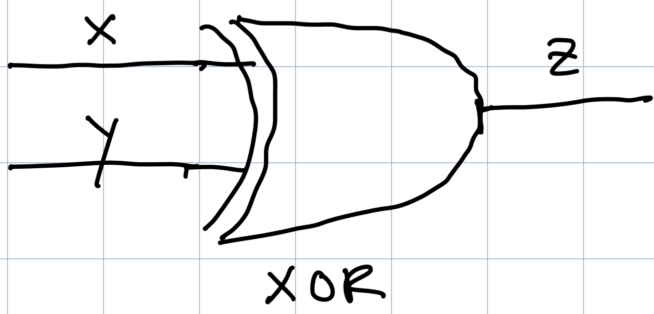


X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1



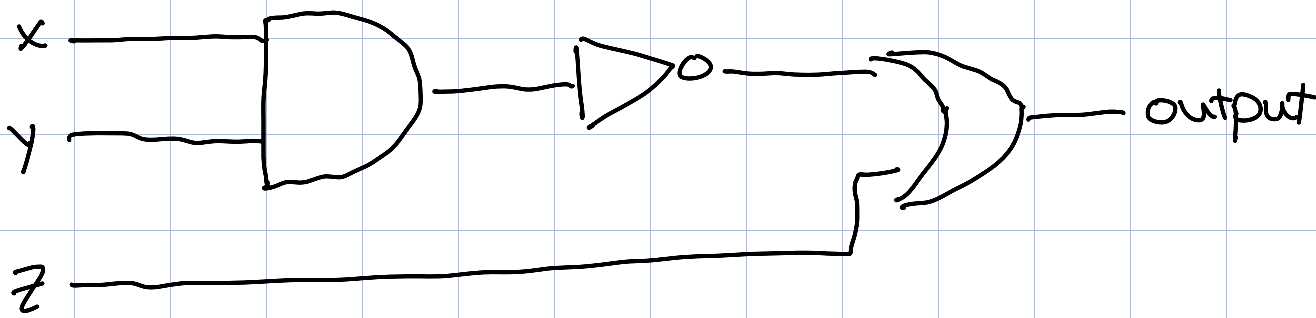
X	Z
0	1
1	0





x	y	z
0	0	0
0	1	1
1	0	1
1	1	0

Circuits are when we connect gates....

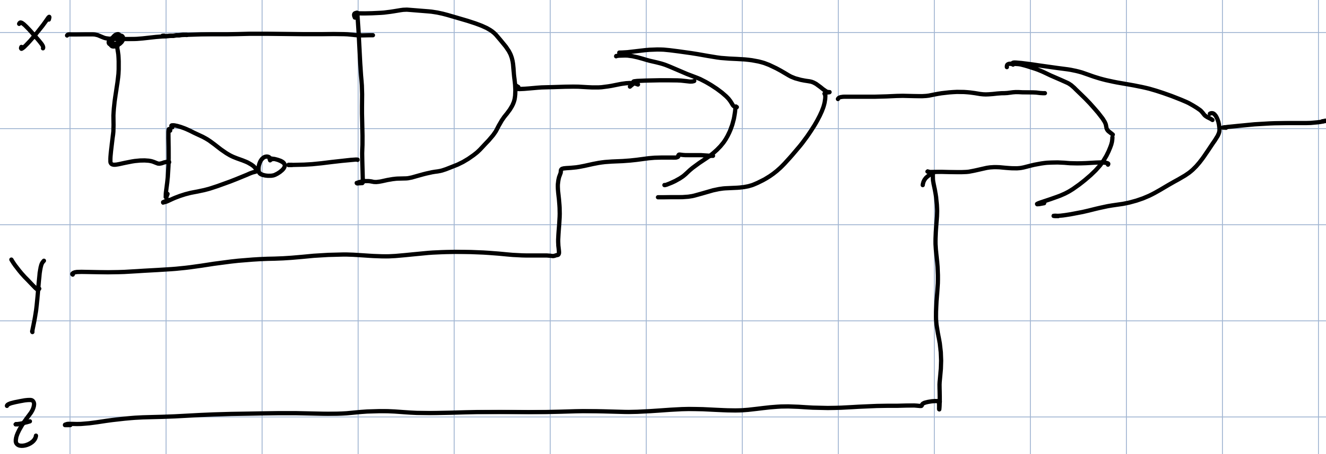


The logic expression above is...

$\neg(x \wedge y) \vee z$

Hint: work left to right when applying operations and remember your ()

Exercise:



1) express using logic

$(x \wedge \neg x) \vee y \vee z$

2) simplify above expression using logic rules

$((x \wedge \neg x) \vee y) \vee z$

$F \vee (y \vee z)$ comp

$y \vee z$ ident

3) Draw simplified expression