1) Admin Exam means funny deadline | Professor Hamlin
includes up to day 10 | Day 9
- Sol'n released for HW 4 on sunday
good news: more time to study
bad news: only may use 1 late day on HW4

2) Review

3) Permutations

4) Counting stragegies
- Partition
- complement
- simplification

Review:

Pigeonhole principle : $\exists$ a pile w/ at least $\lceil N/K \rceil$ items

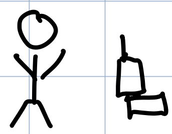
Counting: Product rule (* and *) $*$
Sum rule (* or *) $+$
principle inclusion/exclusion

Exercise: Can wear pants/shirt (AND) ~or~ dress (OR)
(4 pants, 3 shirts, 2 dresses) how many outfits?

$|\text{pants}| * |\text{shirts}| = 4 \cdot 3$
$+$
$|\text{dresses}| = 2$

$\boxed{14}$

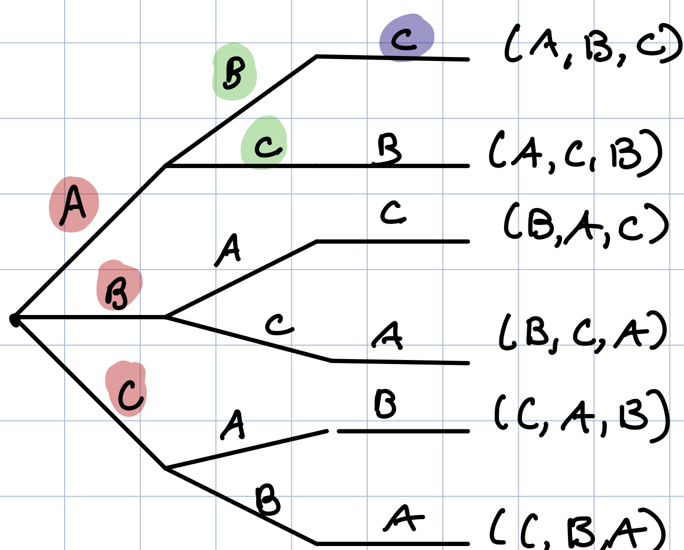
Permutations (order matters)



~~Vaccum Salesman~~ Hamlin Wizard Oil, visiting

Atlanta Boston Chicago

How many ways can the traveling salesman visit the three cities?



Also can think...

3 choices for first city

AND

2 choices for second city

AND

1 choice for third city

we use product rule to combine

|First choice| * |Second choice| * |third choice|

$3 * 2 * 1 = \boxed{6}$

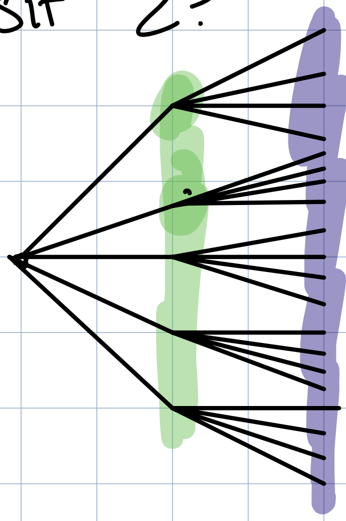
Also known as $\boxed{3!}$ (3 factorial)

Factorial: $n! = n*(n-1)*(n-2)*\ldots 3*2*1$

By convention $0! = 1$

$8! = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

What if we have 5 cities and only visit 2?



5 choices for 1st city

AND

4 choices for 2nd city

$$5 \cdot 4 = 20$$

can also think of it as

$$= \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \frac{5!}{3!}$$

Formally: a permutation is a way of ordering k objects, from total of n

"n permute k"

$$P(n,k) = \frac{n!}{(n-k)!}$$

3 of 3 cities

$n = 3 \qquad k = 3$

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \boxed{3!}$$

$$3 \cdot 2 \cdot 1 = \boxed{6}$$

2 of 5 cities

$n = 5 \qquad k = 2$

$$P(5,2) = \frac{5!}{(5-2)!} = \boxed{\frac{5!}{3!}}$$

$$(5-2)! \neq 5! - 2!$$

Exercise 1) How many ways are there to order 5 people in a line for a photo?

$n = 5$ $\quad k = 5$ $\quad \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$$P(5,5) = \frac{5!}{(5-5)!} = \boxed{5!}$$

2) How many ways of ordering 6 out of 20 people for photo?

$n = 20$ $\quad k = 6$ $\quad \underline{20}\ \underline{19}\ \underline{18}\ \underline{17}\ \underline{16}\ \underline{15}$

$$P(20,6) = \frac{20!}{(20-6)!} = \frac{20!}{14!}$$

Bonus: How big of factorial do you need to 'break' your calculator?

$10! = 3 \times 10^6$

$20! = 2 \times 10^{18}$

Reference #'s

- 10^{17} seconds since big bang
- 10^{80} atoms in universe
- 10^{100} googol

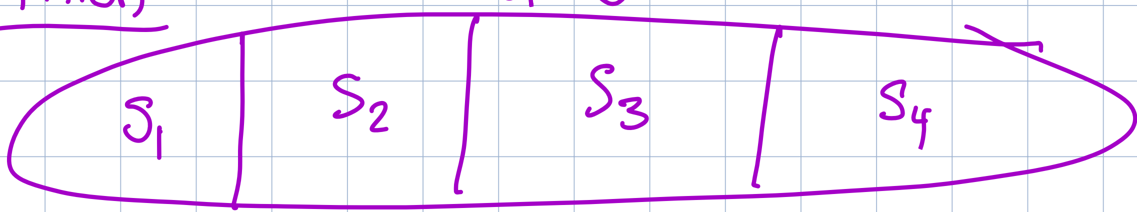
On Hw/Exams - please leave as factorial

e.g $\frac{5!}{2!}$

	replacement	w/o replacement
order matters	password of length 4 (lowercase) $\underline{26}\ \underline{26}\ \underline{26}\ \underline{26}$ 26^4	four people in line for photo $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ $P(4,4)$

Partition

Set S



when we divide larger set into subsets

→ subset are disjoint (overlap)

$$S_1 \cap S_2 = \emptyset$$

→ union of sets is = to original set

$$S_1 \cup S_2 \cup S_3 \cup S_4 = S$$

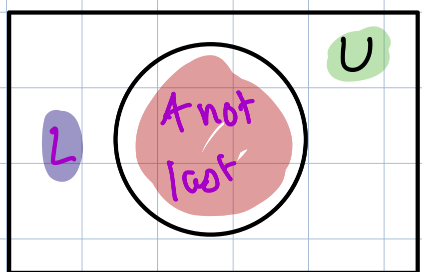
Counting stragetics:

For complex counting problems – how can we approach them?

There are a few common stragetics.

Count-by-Complement

How many ways to order 5 people s.t. A not last?



U = All orderings of 5 people

L = Set of orderings where A is last

idea: if we can calculate (U) we can just subtract all items we aren't interested in (L)

$$|U - L| = |U| - |L|$$

Only works if disjoint!

$|U| =$ $N=5 \quad K=5$

$P(5,5) = 5!$

$$\boxed{\begin{array}{c} |U| - |L| \\ 5! - 4! \end{array}} = 96$$

$N=4 \quad K=4$

_ _ _ _ A

$P(4,4) = 4!$

Count-by-partition

How many ways to order 5 people s.t. A not last?

$n=4 \quad k=4$

A _ _ _ _ $\qquad P(4,4) = 4!$

~or~ $\qquad +$

_ A _ _ _ $\qquad P(4,4) = 4!$

~or~ $\qquad +$

_ _ A _ _ $\qquad P(4,4) = 4!$

~or~ $\qquad +$

_ _ _ A _ $\qquad P(4,4) = 4!$

$\boxed{4 \cdot 4!} = 96$

Split choices into disjoint sets e.g.

$|A \text{ in first}| + |A \text{ in second}| + \ldots + |A \text{ in } 4^{th}|$

Common error is to have overlap between sets - be careful! → if overlap use PIE! and include all options

Count-by-simplification (extension of partition)

How many ways to order 5 people s.t. A not last?

$P(4,4)$

$n=4 \quad k=4$

A ___ A ___ A ___ A ___

ways to order 4 people not A

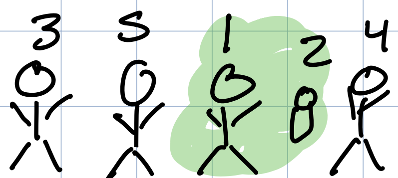
*

4 locations to put

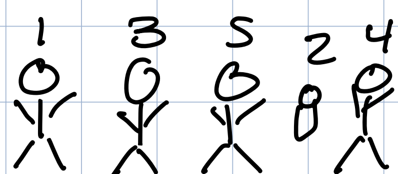
$\boxed{4 \cdot 4!} = 96$

Recognizing that each of you disjoint sets is the same

Example: How many ways to order portrait of 5 people if person 2 is a baby and must be on person 1's immediate right?



Good!



Bad?

Can treat 1+2 as one item

$n=4 \qquad k=4$

$P(4,4) = 4!$

Exercise:

1) How many passwords can be made of lowercase letters no longer than 5 characters? (partition)

letters can be reused

1 char	$\underline{26}$	26
or 2 char	$\underline{26}\ \underline{26}$	26^2
or 3 char		26^3
or 4 char		26^4
or 5 char		26^5

$$26^5 + 26^4 + 26^3 + 26^2 + 26$$

2) How many length 4 passwords (lowercase) start w/ a or end w/ b

(partition but be careful!)

a _ _ b

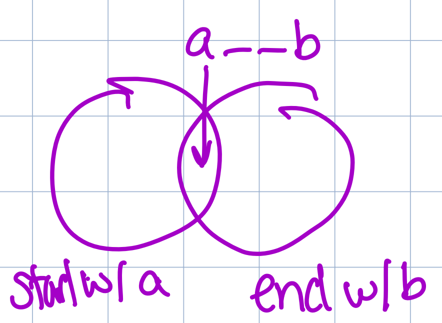
starts w/a

a _ _ _ $\quad 26^3$

end w/b

_ _ _ b $\quad 26^3$

$$\boxed{(26^3 + 26^3) - 26^2}$$



Exercise:

1) How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

$|U| - |L|$

$|U|$ = passwords length 10 $\qquad n^k$

$\underline{26}\ \underline{26}\ \_\ \_\ \_\ \_\ \_ = 26^{10}$

$|L|$ = start w Qwerty

QWERTY _ _ _ _ $= 26^4$

$$\boxed{26^{10} - 26^4}$$

2) How many ways are there to order 3 people in a wedding photo for romeo and juliet?
Assume:

- there are 10 Montague's (Romeo's family, excluding him) who could be in the photo
- there are 7 Capulets (Juliet's family, excluding her) who could be in the photo
- Romeo and Juliet are too busy dancing to be in any picture
- Montagues and Capulets won't get in the same photo (that whole Tybalt / Mercutio thing...)

(hint: partition, [illegible])

Photo of M	Photo of C
$N=10 \quad K=3$	$N=7 \quad K=3$
$P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!}$	$P(7,3) = \frac{7!}{(7-3)!} \to \frac{7!}{4!}$

$$\boxed{\frac{10!}{7!} + \frac{7!}{4!}}$$

3) How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?

[redacted] complement then partition

$\underline{5}\ \underline{2}\ \underline{3}\ \underline{1}\ \underline{4}$ valid

$\underline{5}\ \underline{2}\ \underline{1}\ \underline{3}\ \underline{4}$ invalid

$|U| =$ order 5 of 7 $\quad n=7\ k=5 \quad P(7,5)$

$|L| =$ 2 right next to 1

$\underline{2}\ \underline{1}\ \_\ \_\ \_ \qquad P(5,3)$

$\_\ \underline{2}\ \underline{1}\ \_\ \_ \qquad P(5,3)$

$\_\ \_\ \underline{2}\ \underline{1}\ \_ \qquad P(5,3)$

$\_\ \_\ \_\ \underline{2}\ \underline{1} \qquad P(5,3)$

$$P(7,5) - 4 \cdot P(5,3)$$

$$\frac{7!}{2!} - 4 \cdot \frac{5!}{2!}$$

Another approach

1 included or 1 is not included

complement