Professor Hamlin
Day 6

Agenda

1) Review
2) Sets
   - vocab
   - builder notation
   - set operations → union, intersection, complement, difference

Review

Extended Conditionals

contrapositive, inverse, converse

double implication

negating cond. $\neg(x \to y) = x \wedge \neg y$

Extended Quantifiers

negating Quant.

nested quantifiers

Exercises:

1) Given the following conditional identify these variants: "If I am sleepy then I nap" $x \to y$

a) If I'm not sleepy then I don't nap. $\neg x \to \neg y$

inverse

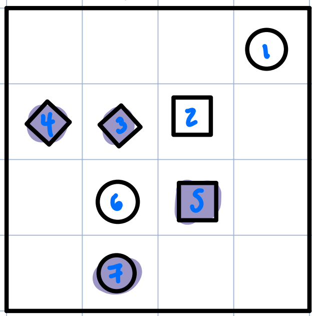
b) If I don't nap then I'm not sleepy $\neg y \to \neg x$

contrapos.

c) If I nap then I am sleepy and if I'm sleepy I nap

$\underbrace{y \to x}_{\text{converse}} \wedge x \to y$] double imp.

2) Tarski World



State if the following are true/false
if false provide counterexample

a) $\forall x \exists y$: circle(x) ^ square(y) ^ x Rightof y (x,y)

all circles have some square

False, 6,7 has no square left of it.

b) $\exists x \forall y$: circle(x) ^ square(y) ^ x is Abovey (x,y)

true

Sets

So far what x/y/z etc have been has been confusing

$\forall x$: Yellow(x)

is x a pet, a student an object?

Need a way of defining what x is → as well as talking about groups in general

Sets

Set: a collection or group of unique items

{1, 2, 3} or {a, b, c}

{1, 2, 3, a, b, c}

order doesn't matter

Notation note: {1, 2, 3, 4} = {1, 2, 3, 4, 4, 4} = {2, 3, 1, 4} these two are equivalent but it is bad form to have an object show up more than once

Common sets:

- $\emptyset = \{\}$ empty set w/ no items

  an object that might show up in another set

$\mathbb{Z}$

- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2 \ldots\}$ integers

  sets can be infinite!

$\mathbb{N}$

- $\mathbb{N} = \{0, 1, 2, 3 \ldots\}$ natural numbers

  ↑ sometimes excluded

- $\mathbb{R} = \{-2, 0, 1\frac{1}{2}, \pi \ldots\}$ Real numbers

Set membership

An object is either in or out of a set

boolean statement about x and its presence in the set

$x \in \mathbb{Z}$ — x in the integers

$2 \in \mathbb{Z}$ = T

x is used to denote object.

$x \notin \mathbb{N}$ — x not in the natural #s

$-2 \in \mathbb{N}$ F

These are both boolean statements – either true or false

Exercise True or false

1) $0 \in \mathbb{R}$ T
2) $-1 \notin \mathbb{N}$ T
3) $0 \in \emptyset$ F
4) $2.5 \in \mathbb{Z}$ F
5) $5 \in \mathbb{N}$ T
6) $-\pi \notin \mathbb{R}$ F

Set builder notation

Listing out items, or if there is no easy pattern like the reals, is hard. Solution? Set builder notation.

$$A = \{ x \in \mathbb{N} \mid (3 \leq x) \wedge (x \leq 5) \}$$

s.t.

A is the set of x in the natural numbers such that x is greater or equal to three and less than or equal to five

The construction is :

$$A = \{ x \in \text{Some larger Universe} \mid \text{Some predicate about } x \}$$

if the predicate is T, x is in the set A

F, x is not in the set A

Ex : $\{ x \in \text{CS1800students} \mid \text{Cool}(x) \}$

The set of all students s.t. the student x is cool

Exercise

1) List all items in this set

$A = \{ x \in \mathbb{Z} \mid |x| < 5 \}$ absolute value

$\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

2) Write w/ set builder notation

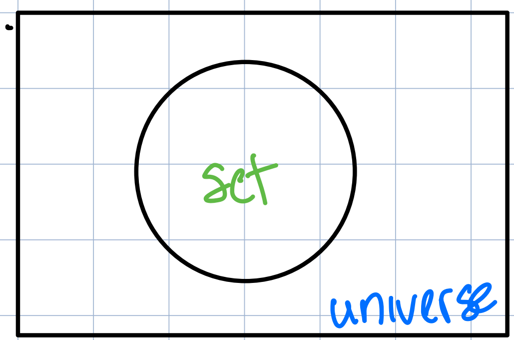
B = set of natural # x s.t. (x mod 3 = 0 and x mod 7 = 0 and x < 40)

$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \wedge (x \bmod 7 = 0) \wedge (x < 40) \}$

3) $2 \in \{ y \in \mathbb{Z} \mid y > 4 \}$? F

Venn diagrams:

A way of visually representing sets



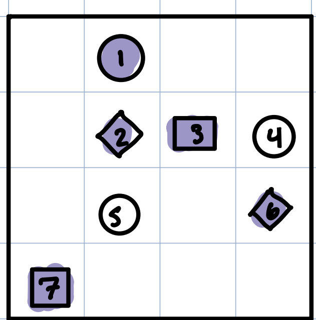
universe: larger set e.g. all things in the universe, students at NEU

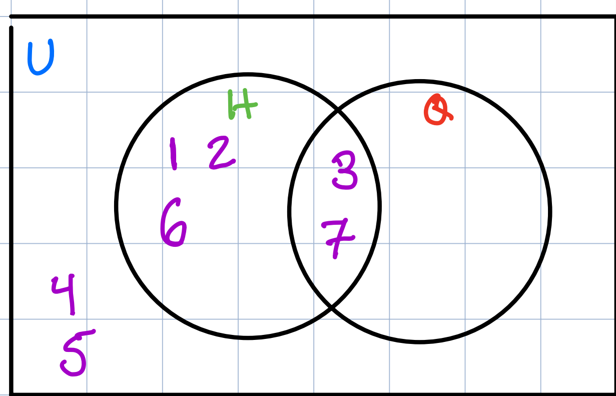
Example

U = all shapers

H = shaded shapes

Q = squares





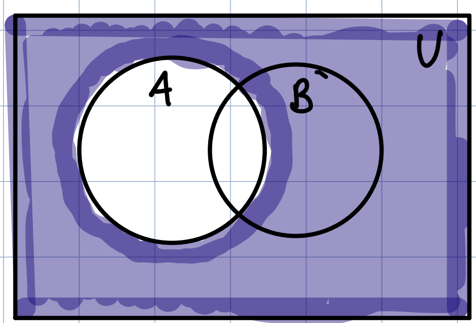
Note: just because an area exists doesn't mean it has items in it!

Set operations:

Logic Operators (on predicate)	NOT	AND	OR
Set operators	Complement	intersection	union

Complement: $\bar{A} = A^c = \{x \in U \mid x \notin A\}$

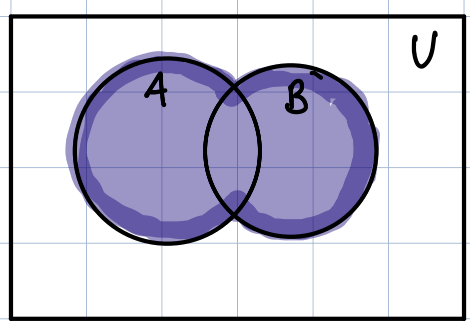
All x in universe s.t. x is not in A



applying NOT to predicate

Union: $A \cup B = \{x \in U \mid x \in A \vee x \in B\}$

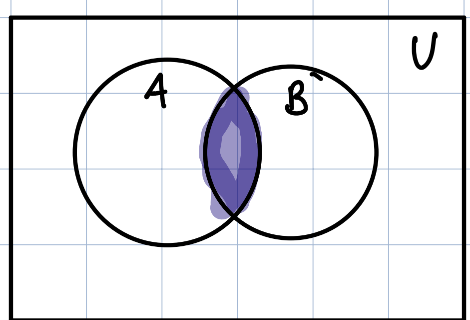
All x in universe s.t x in A or x in B.



$\boxed{\cup \text{ - union}}$

Intersection: $A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$

A x in universe s.t. x in A and x in B

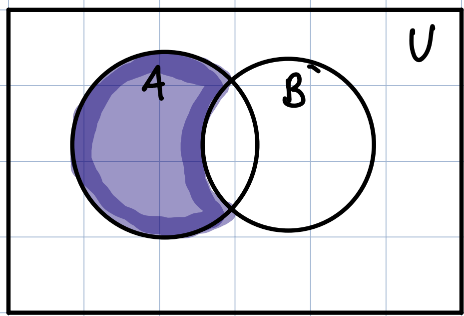


$A \cap B = B \cap A$

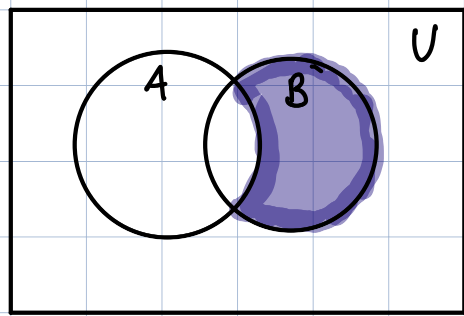
Difference: $A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$

all items in one set (A) not the other (B); order matters!

A − B

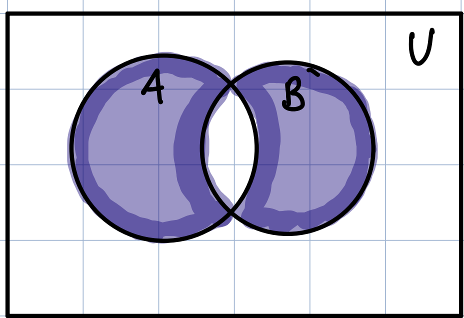


B − A



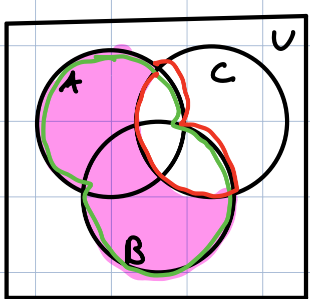
Symmetric Difference: $A \Delta B = \{x \in U \mid x \in (A \cup B) \wedge x \in (A \cap B)\}$

all items in one set or other, not both (XOR)

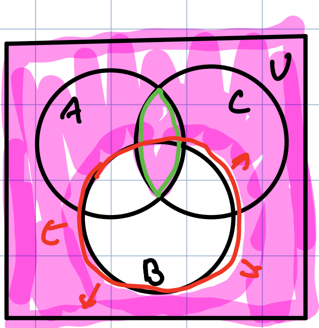


x	y	x xor y
F	F	F
T	F	T
F	T	T
T	T	F

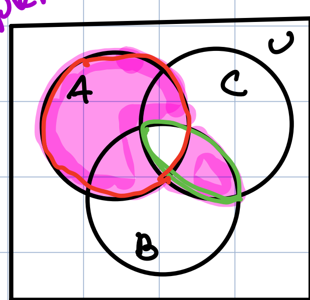
Exercise: Shade areas for venn diagram



$(A \cup B) - C$ diff.



$(A \cap C) \cup \overline{B}$ complement



$A \Delta (B \cap C)$ sym. diff.

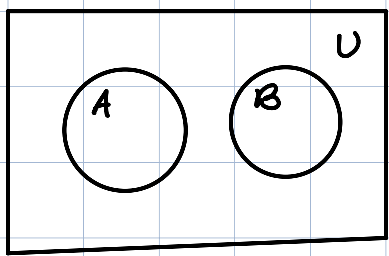
Set membership $x \in A$

Set Relationships

oper $\cup, \cap, -, \Delta, {}^c$

$1 = x+3$, $<$, $>$, $\leq$, $\geq$

Disjoint set: A & B are disjoint if no item is in both sets. OR $A \cap B = \{\} = \emptyset$

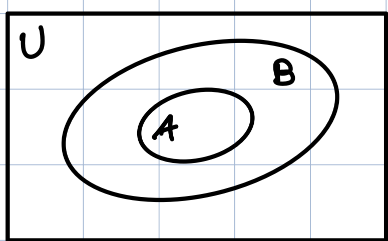


Subset: A is subset of B if All items in A are also in B

$\subseteq$ = contain by

$A \subseteq B = x \in A \to x \in B$

if x in A then x in B



Set Equality: A = B if A is subset of B and B is subset of A

$2 = 2$
$2 \leq 2$
$2 \geq 2$

all elements in A & B are the same,

predicates are logically equivalent

Strict (Proper) Subset $\subset$

$x \leq 3$ includes 3

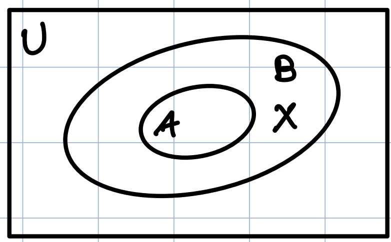
$x < 3$ does not include 3

same for sets → want to say set is strictly smaller

$A \subseteq B$ A might equal B

$A \subset B$ B has items A does not

$A \subset B$ = All items in A also in B AND B contains an item not in A $\quad A \subseteq B \wedge B - A \neq \emptyset$



Note: only use $\subseteq$, $\subset$ with sets on both sides NOT $\in$

<u>Exercise:</u> $A = \{1, 3, 5, 7\}$, $B = \{x \in \mathbb{Z} \mid 1 \le x < 10\}$

$C = \{2, 4, 6, 8\}$

1) $5 \in A \cap B$ T
2) $A \subset B$ T
3) $B \subseteq B$ T
4) $C \in B$ wrong notation
5) $B \subset A \cup C$ F, missing 9
6) $C \subset B - A$, T, 9 is in $B-A$ not C

Note $\emptyset \subseteq A$ or $\emptyset \subset A$ is always true

Other Terminology

Cardinality: number of unique items in set

$|A|$ = cardinality of set A

Ex: $A = \{1, 2, 3, 4\}$ $\quad B = \emptyset$ $\quad C = \{\emptyset\}$

$|A| = 4$ $\quad |B| = 0$ $\quad |C| = 1$

Sets can contain items that are other sets!

$D = \{\{1,2\}, 3, 4\}$ $\quad$ $\mathcal{E} = \{\{1,2,3,4\}\}$

$|D| = 3$ $\quad$ $|E| = 1$

Powerset: The <u>set of sets</u> that can be made with orignal set

$A = \{1,2\}$ $\quad$ $\mathbb{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

Another way to think about it . . .

$\mathbb{P}(A) = \{$

1	2	
F	F	$\emptyset$
F	T	$\{2\}$
T	F	$\{1\}$
T	T	$\{1,2\}$ $\}$

Exercise: $A = \{1, 5, 7\}$

1) $|A| = 3$ $\quad$ 2) $|\mathbb{P}(A)| = 8$

1	5	7	
F	F	F	$\{\emptyset$
F	F	T	$\{7\}$
F	T	F	$\{5\}$
F	T	T	$\{5,7\}$
T	F	F	$\{1\}$
T	F	T	$\{1,7\}$
T	T	F	$\{1,5\}$,
T	T	T	$\{1,5,7\}\}$