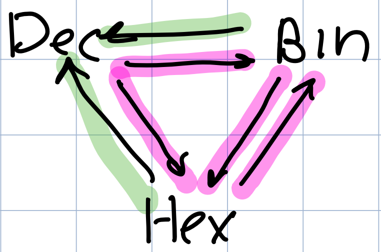
Agenda

1) Review

🟢 = last class

🟣 = today

2)



3) adding / mult in another base

3) modular arithmetic 🕐

What did we cover last time?

- Bases (Binary / Hex)

→ Bases into decimal

1. canonical rep

$1 \cdot 1000_2 + 0 \cdot 100_2 \cdots$

2. convert 2/16 #'s to decimal

$1000_2 \to 2^3 \to 8$

3. Add it all up

Practice

1) $1011_2 = 1 \cdot 1000_2 + 0 \cdot 100_2 + 1 \cdot 10_2 + 1 \cdot 1_2$

$1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

$\boxed{11}$

2) $123_4 = 1 \cdot 100_4 + 2 \cdot 10_4 + 3 \cdot 1_4$

$$1 \cdot 16 + 2 \cdot 4^1 + 3 \cdot 4^0$$

$$\boxed{27}$$

Decimal to other bases

14_{10} in base 2

Remember powers of 2

$2^0 = 1 = 1_2$

→ $2^1 = 2 = 10_2$

→ $2^2 = 4 = 100_2$

→ $2^3 = 8 = 1000_2$

→ $2^4 = 16 = 10000_2$

How many times does 16 go in? 0 $\boxed{0 \cdot 10000_2}$

.... 8? 1 $\boxed{1 \cdot 1000_2}$

But we now need to handle only the remaining value - we have handled 8.

$14 - 8 = 6$

. . . 4? 1 $\boxed{1 \cdot 100_2}$

Handle only remaining value $6 - 4 = 2$

. . . 2? 1 $\boxed{1 \cdot 10_2}$

Remaining value $2 - 2 = 0$

. . . 1 ? 0 $\boxed{0 \cdot 1_2}$

Remaining value $0 - 0 = 0$

All together . . .

coefficients

$$0 \cdot 10000_2 + \underbrace{1} \cdot 1000_2 + \underbrace{1} \cdot 100_2 + \underbrace{1} \cdot 10_2 + \underbrace{0} \cdot 1_2$$

$$\cancel{0}\ \cancel{0}\cancel{0}\underbrace{1110}_2$$

Practice 13_{10}

2^3 gointo 13	$1 \cdot 1000_2$	R 5
2^2 go into 5	$1 \cdot 100_2$	R 1
2^1 go into 1	$0 \cdot 10_2$	R 1
2^0 go into 1	$1 \cdot 1_2$	R $\underline{0}$

$\boxed{1101_2}$

We can also think about doing this backwards if we don't have our handy table of powers

Euclid's Division

(Subtraction)
First method - start w/ largest power of two and work down

Euclid's - start w/ smallest power and get bigger

$$\begin{array}{r} 7 \\ 2\overline{)14} \end{array} \text{ R } 0$$

Recipe: (okay to just memorize!)

0. start w/ decimal value
1. Divide value w/ base get remainder + multiplier
2. Stop if multiplier is 0 otherwise continue step 1
3. Read remainders BOTTOM TO TOP

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$\boxed{1110_2}$$

Practice: Express 23_{10} as binary using
1. Subtraction
2. Euclid's

Extra: How are these methods similar / different?

1)

16 go 23	1·16 R7
8 go 7	0·8 R7
4 go 7	1·4 R3
2 go 3	1·2 R1
1 go 1	1·1 R0

$$\boxed{10111_2}$$

2)

$$23 = 11 \cdot 2 + 1$$
$$11 = 5 \cdot 2 + 1$$
$$5 = 2 \cdot 2 + 1$$
$$2 = 1 \cdot 2 + 0$$
$$1 = 0 \cdot 2 + 1$$

$$\boxed{10111_2}$$

Note Euclid works for other bases as well!

$23_{10} \rightarrow$ base-6

$$\begin{aligned} 23 &= 3\cdot 6 + 5 \\ 3 &= 0\cdot 6 + 3 \end{aligned} \uparrow$$

$\boxed{35_6}$

Math in other bases

Addition

$$\begin{array}{cccc} & 1 & & \\ & 1 & 2 & 3 \\ + & 2 & 8 & 1 \\ \hline & 4 & 0 & 4 \end{array}$$

$$\begin{array}{cccc} & 1 & & \\ & 3 & C & 4_{16} \\ + & 1 & 5 & 2_{16} \\ \hline & 5 & 1 & 6_{16} \end{array}$$

What is C?

$A = 10$

$B = 11$

$C = 12$

$12_{10} + 5_{10} = 17_{10}$

$16_{10} + 1_{10}$

11_{16}

Multiplication

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & 2 & 3_{10} & \\ \times & & 4 & 1_{10} & \\ \hline & 1 & 2 & 3 & \\ + & 4 & 9 & 2 & 0 \\ \hline & 5 & 0 & 4 & 3 \end{array}$$

$$\begin{array}{cccc} & 1 & & \\ & 1 & 7 & 2_8 \\ & \times & 2 & 1_8 \\ \hline 1 & {}^{1}1 & 7 & 2 \\ 3 & 6 & 4 & 0 \\ \hline 4 & 0 & 3 & 2 \end{array}$$

Remember all the digits are already base 8

$7_{10} \times 2_{10} = 14_{10}$

$7_{10} + 4_{10} = 11_{10} \rightarrow$

$$\begin{aligned} 14 &= 1\cdot 8 + 6 \\ 1 &= 0\cdot 8 + 1 \end{aligned} \uparrow$$

→ If stuck try a similar decimal problem

Practice

1)
$$\begin{array}{r} \scriptstyle 1 \; 1 \;\; \\ 1\,4\,7_8 \\ +\;\; 4\,4_8 \\ \hline 2\,1\,3_8 \end{array}$$

$9 = 1\cdot 8^1 + 1\cdot 8^0$

2)
$$\begin{array}{r} \scriptstyle 1 \;\;\; \\ 3\,2_4 \\ \times\;\; 2\,2_4 \\ \hline {\scriptstyle 1}\;1\,3\,0 \\ +\;1\,3\,0\,0 \\ \hline 2\,0\,3\,0_4 \end{array}$$

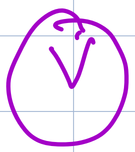
$4_{10} \rightarrow 10_4$

$7_{10} \rightarrow 13_4$

$4+3$

Modular Arithmetic

Who here has ever done modular arithmetic?

You all have  clock math

Currently 9 PM if I start a 4 hr movie marathon what time will I go to bed?

9 + 4 = 1 AM (not military time)

9 PM + 2 hrs = 11 PM

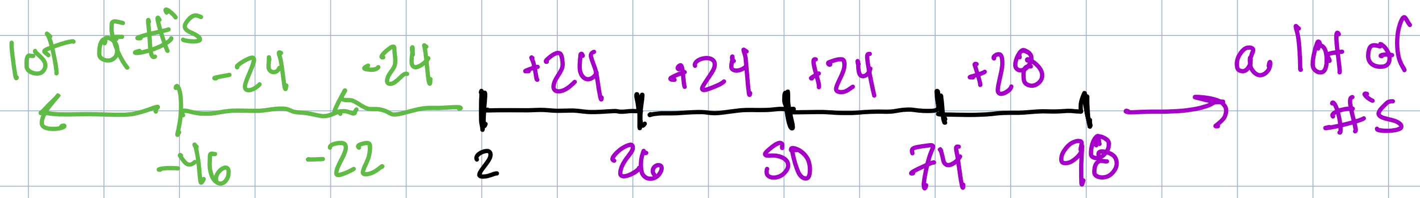
9 PM + 26 hrs = 11 PM

9 PM + 50 hrs = 11 PM

times are equivalent if they differ by factor of 24

Modulo: an intuition

What are the numbers which add 2 hrs to 9?



↑ this represents all these #s

all of these are 2 mod 24

x mod n

A definition: modulo is the remainder when you divide the number by the mod

98 mod 24
= 2

$$\boxed{24\,\overset{4}{\overline{)98}}\quad R\ 2}$$

Practice

1) 15 mod 4 = 3 $\quad 4\,\overset{3}{\overline{)15}}\quad R\ \boxed{3}$

2) 14 mod 7 = 0 $\quad 7\,\overset{2}{\overline{)14}}\quad R\ \boxed{0}$

Negative Numbers? Same idea w/ one more step

-28 mod 5

$$\begin{array}{r} 5 \\ 5\overline{)28} \end{array} \text{ R } 3$$

But the remainder is actually negative!

-3 mod 5

Just add mod value

$-3 + 5 = 2$

Unary
Base - 1

0 00 000

number is number of zeroes

Practice

1) -17 mod 3

$$\begin{array}{r} -5 \\ 3\overline{)-17} \end{array} \text{ R } -2$$

-2 mod 3 = 1

-2 + 3

Extra 2) if $3 = x \bmod 4$ and $3 < x \leq 15$ what values work for x

Hint what values equal 3 mod 4? list them out

$$3 \xrightarrow{+4} \underline{7} \xrightarrow{+4} \underline{11} \xrightarrow{+4} \underline{15} \xrightarrow{+4} 19$$

7, 11, 15