Clearly print below (to be read by a computer):

Name:

CS1800 Discrete Structures	Prof Higger & Prof Hamlin
Fall 2024	Nov 2024

Exam2 (Practice)

Due: do-not-submit

Exam Instructions:

- **Please write your name & NUID in the box on top of this page**
- This exam is a strictly individual effort, you may not view or share exam content (including discussing exam questions with other students) until grades have been released.
- Unless otherwise specified, be sure to show your detailed work to earn full credit
- You are welcome to use a calculator without any communication abilities (e.g. no cellphones, or wifi / bluetooth enabled calculators).
- To keep the exam fair to all students, no math-content related help will be offered to students. We are, however, happy to clarify any instructions but please don't expect any hints.
- Your work must be easiliy legible to be graded.
- When the final exam time has been called, please promptly turn in your work. Exams which are submitted late may be penalized 10 points per minute late (in practice we are not likely to apply this penalty so long as you stop working and get up to hand in your exam when time is called).

- $E[x] = \sum_x x * P(X = x)$
- $\texttt{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$

	No Repeat Selections	Repeat Selections
Order Matters	Permutations: $P(n,k) = \frac{n!}{(n-k)!}$	Product Rule: n^k
Order Doesn't Matter	Combinations: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$	Stars and Bars (Partitions of k identical objects into n groups): $\binom{k+n-1}{n-1}$

Note

- The instructions above are identical to exam2's, they're included for your reference.
- Many of these problems are redundant with each other, but please don't be intimidated by having so many problems here on the practice exam, the real exam is certainly shorter! If you feel confident with the first problem of some type, feel free to skip the second.
- We have an equality and an inequality style induction problem here. On the exam you'll only have one or the other.

Happy studying :)

Problem 1 Counting Cards

Given a standard 52-card deck of playing cards (half of cards are red and half are black) one draws 5 cards without replacement. What is the probability that the first three cards drawn are red, and the last two cards drawn are black? (Notice: this event does not include drawing two black cards before then three red cards).

Be sure to show and justify and clearly document your work as well as computing a final probability value.

Problem 2 Expected Value & Variance: Call Center Wait Times

An internet service provider's call center receives calls which are each one of the following types:

Call Type	Time of Call (min)	Probability
General Inquiry	2	0.20
Technical Support	4	0.25
Billing Questions	3	0.15
Service Activation	7	0.10
Device Troubleshooting	5	0.30

Table 1: Probability of Response Time for Call Center (Sum to 1)

i Compute the expected time it takes to handle a single call

ii Compute the variance in time it takes to handle a single call

iii An employee working at the center is able to handle 15 calls in an hour (this employee's calls follow the same distribution as above). Is this employee typically faster or slower than others in the group? Explain

Problem 3 Expected Value & Variance: Basketball

A basketball player makes shots according to the following table:

	Distance to net	**Points earned (if made)**	**Prob shot made (%)**
P	In Paint	2	45
J	Mid Range Jump	2	40
T	3 pt	3	35

Where we use random variables P, J or T to indicate the number of points earned when a player takes each of the corresponding shots.

i Compute the expected value and variance of: P

ii Compute the expected value and variance of: J

iii Compute the expected value and variance of: T

iv If your team was down by 1 point with time for 1 more shot, which shot should this player prefer to maximize their chance of winning?

v In the early game, which shot should this player prefer?

Problem 4 When it rains, park wherever you'd like

A license plate reader is a system which identifies the text of a license plate given a camera's image of the back of a car (useful in parking enforcement). When its not raining, the system is able to identify 99% of the license plates correctly. Unfortunately, when it rains the system struggles to see clearly and only identifies 90% of the license plates correctly. In a particularly rainy parking lot, there is a $\frac{1}{4}$ chance of rain each day.

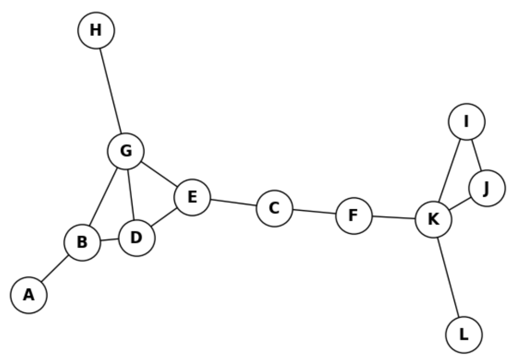
i What is the probability that its raining and a license isn't read correctly?

ii What is the probability that its not raining and a license isn't read correctly?

iii What is the probability that a license isn't read correctly?

iv Suppose that you receive a parking ticket for someone else's car because their license plate was incorrectly read as yours. What is the probability that it was raining when this other car's license plate was read incorrectly?

Problem 5 Breadth / Depth First Search



Wherever possible below, select the node which is earlier in the alphabet first (e.g. prefer visiting node A first over node B, when the search allows you to visit either).

i Starting at L, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

ii Starting at A, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

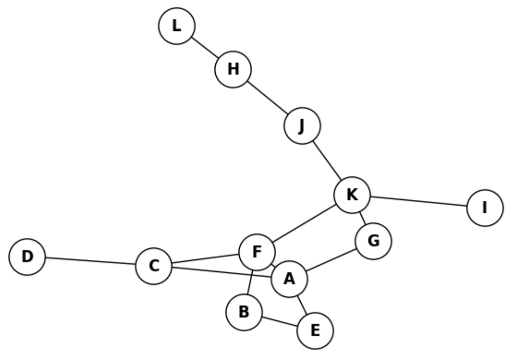
iii Starting at D, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

iv Starting at L, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

v Starting at A, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

vi Starting at D, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

Problem 6 Breadth / Depth First Search



Wherever possible below, select the node which is earlier in the alphabet first (e.g. prefer visiting node A first over node B, when the search allows you to visit either).

i Starting at L, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

ii Starting at D, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

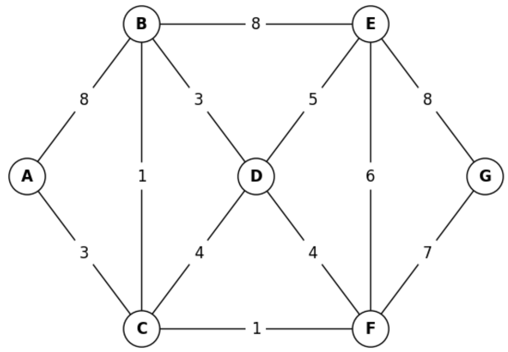
iii Starting at E, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

iv Starting at L, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

v Starting at D, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

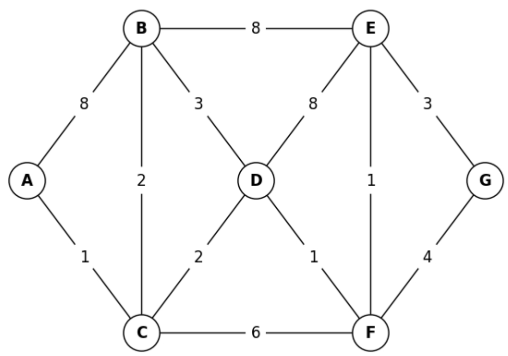
vi Starting at E, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

Problem 7 Dijkstra's Shortest Path



Using Dijkstra's algorithm, find the shortest path from node A to G. Please provide a table which shows the path weight and predecessor from A to every node, labelling the visited node at each step. an example solution is given here.

Problem 8 Dijkstra's Shortest Path



Using Dijkstra's algorithm, find the shortest path from node A to G. Please provide a table which shows the path weight and predecessor from A to every node, labelling the visited node at each step. an example solution is given here.

Problem 9 Induction (equality): $\sum k^2$

The sum of the squares of the first n positive integers may be computed directly

$$1^2 + 2^2 + \ldots + n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Prove the equality above using induction.

Problem 10 Induction (inequality): $n! < n^n$

Using induction, show that $n! < n^n$ for all $n = 2, 3, 4, \ldots$.