CS1800 Day 9

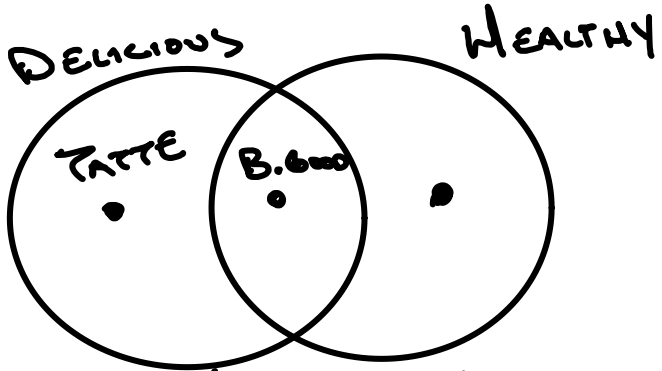
Admin (we'll start @ 9:52)
- exam1 is on Oct 15th
    - Fri Oct 11: "practice" exam together in class and work through any tough examples
- hw3 due today, hw4 released today
- hw4 deadlines are funny (for exam):
    - includes content from day10 (next class)
    - solutions for hw4 released sunday oct 13 @ 12:01am, first thing in the morning
        - good news: allows you study
        - bad news: you may only use up to 1 late day on hw4

Content:
- review PIE & product rule
- permutations
- count by partition
- count by complement
- count by simplification

Reminder: Sum Rule & Principle of Inclusion-Exclusion (Sum rule is a special case of PIE)

When counting the ways we can select one item from either of two sets:

How many different places could I go to for lunch at Northeastern?



$$|D \cup H| = |D| + |H| - |D \cap H|$$
$$= 2 + 2 - 1 = 3$$

Reminder: Product Rule (counting Cartesian Products)

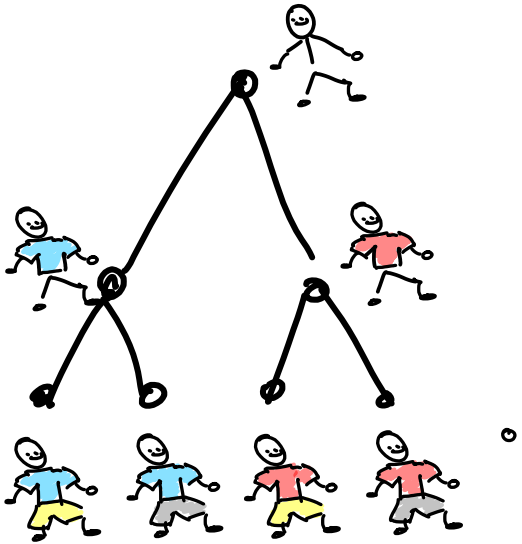
When counting the ways we can select an item from one set AND an item from another set:

How many different outfits can be formed?

$$|S \times P| = |S| \times |P| = 2 \cdot 2 = 4$$

$S = \{$  $\}$

$P = \{$  $\}$



Notation (Reminder)

Set

$\{a, b, c\}$

No repeats

Unordered

Tuple

(a, b, c, a)

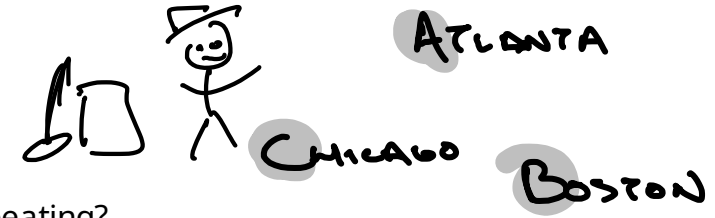
May repeat

Order matters

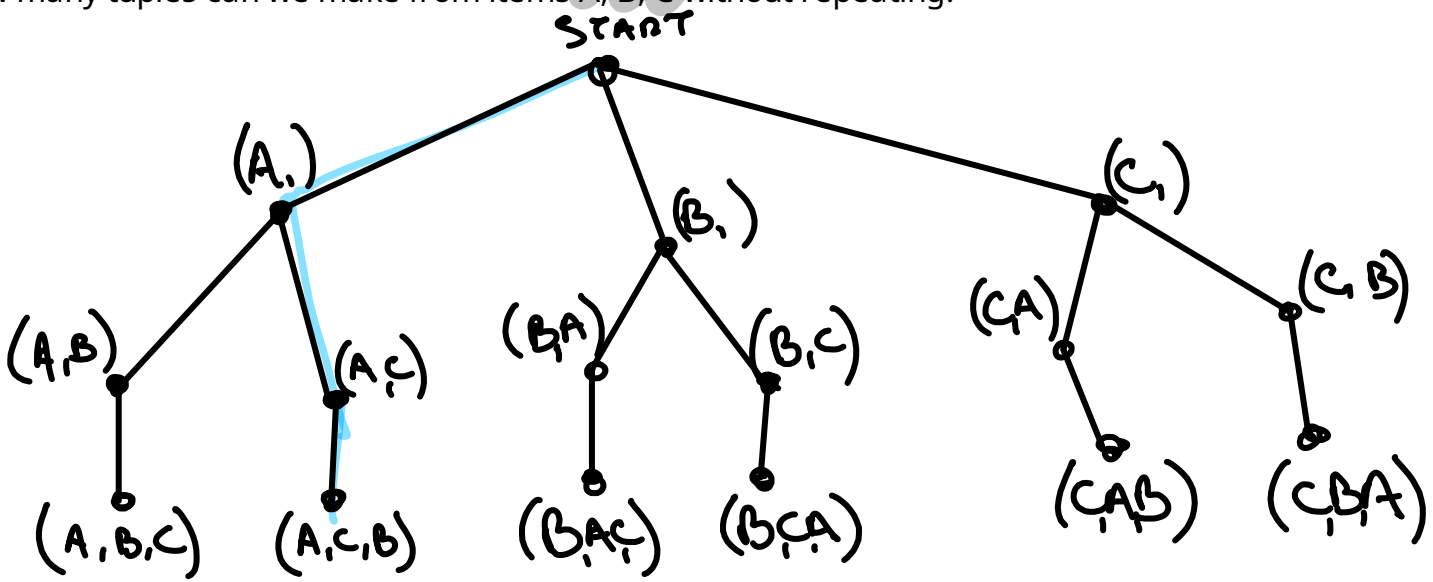
$(a, b) \neq (b, a)$

Permutations: Travelling Salesperson

How many ways can a salesman order 3 city visits?



(How many tuples can we make from items A, B, C without repeating?



Permutations: Travelling Salesperson

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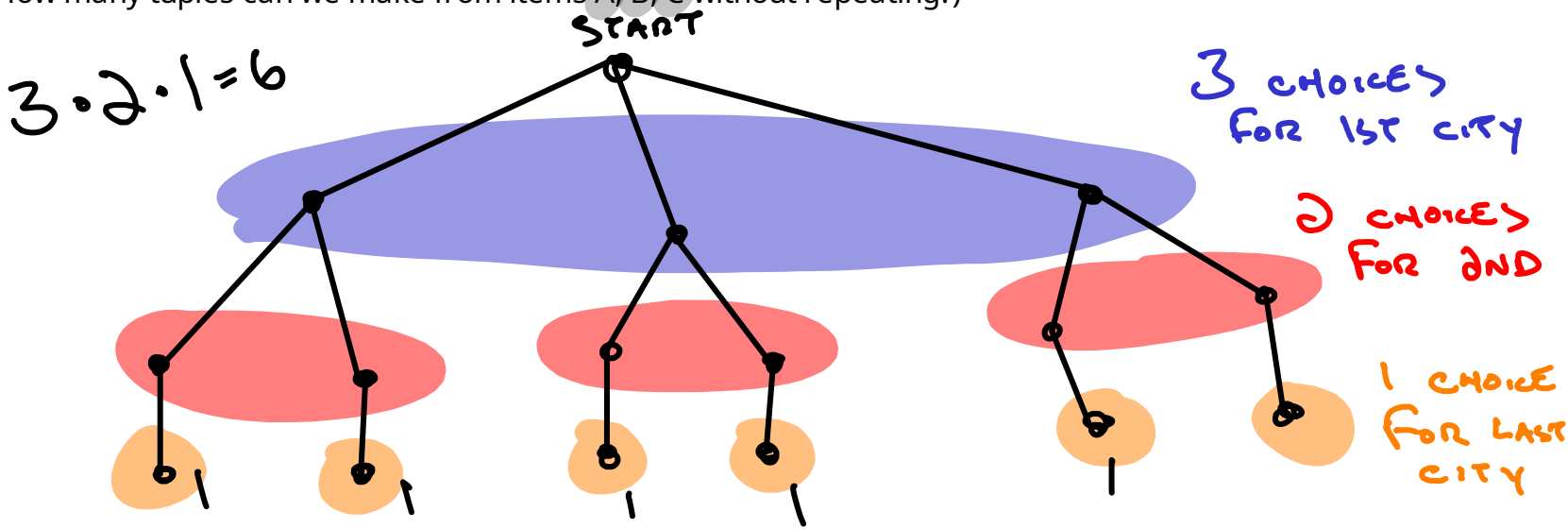


Factorial:

"8 Factorial"

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Convention $0! = 1$

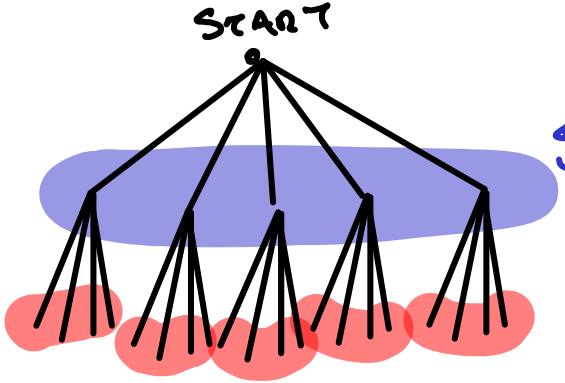
Our salesman (of previous slide) had

$3! = 3 \cdot 2 \cdot 1$ total orderings of 3 cities

Permutations: A Travelling (lazy) Salesperson

How many ways can a salesman order 2 of 5 cities?

(How many tuples of length 2 can be made from A, B, C, D, E where no repeats allowed)?



5 CHOICES FOR 1ST CITY

4 CHOICES FOR 2ND CITY

$$5 \cdot 4 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{5!}{3!}$$

Permutations:

The number of ways of ordering k objects, from n total available is:

$$P(n,k) = \frac{n!}{(n-k)!}$$

$$P(5,2) = \frac{5!}{3!} = 5 \cdot 4$$

$n = 5$

$k = 2$

Previous Examples

Visit 3 of 3 cities

$$P(3,3) = \frac{3!}{(3-3)!} = 6$$

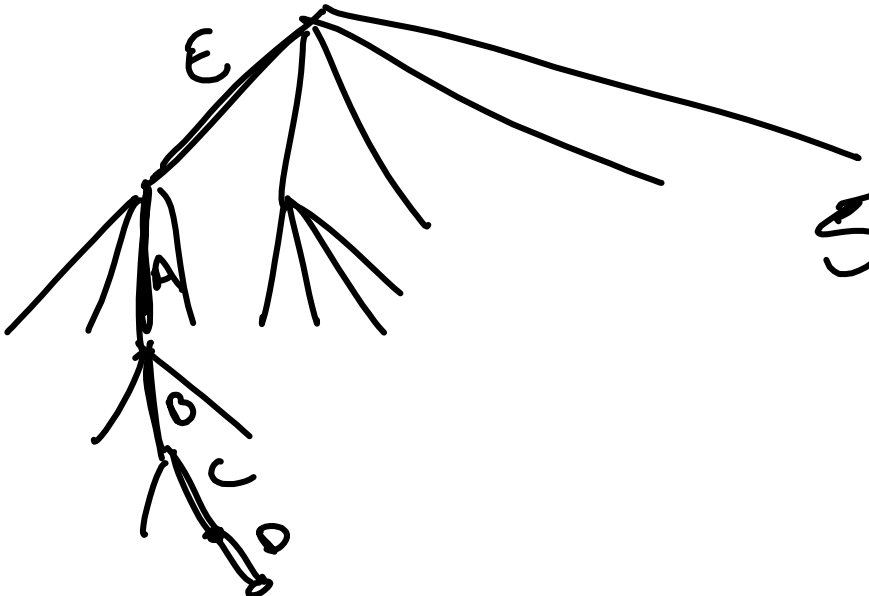
Visit 2 of 5 cities

$$P(5,2) = \frac{5!}{(5-2)!} = 20$$

In Class Activity

A B C D E

How many ways are there to order 5 people for a family portrait?



$5!$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

E A B C D

A E B C D

In Class Activity

How many ways are there to order 6 of 20 people for a family portrait?

$$P(20,6) = \frac{20!}{(20-6)!} = \frac{20!}{14!} = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15$$

In Class Activity

How many ways are there to order 5 people for a family portrait?

How many ways are there to order 6 of 20 people for a family portrait?

(If time): Plug a few of these factorials into calculator or google, how big of a factorial do you need to plug in until you "break" your computer?

$1! = 1$

$2! = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

Factorials grow really quickly: (more on this when we study "function growth" later)

$10! \approx 3.6 \cdot 10^6$ 3.5 MILLION

$20! \approx 2 \cdot 10^{18}$ 2 MILLION BILLIONS

$19! \approx 10^{17}$ SECONDS SINCE BIG BANG

$50! \approx 10^{80}$ ATOMS IN UNIVERSE

$70! \approx 10^{100}$ GOOGLE

Convention (in this class):

FEEL FREE TO LEAVE EXPRESSION AS

$P(5,3)$ OR $\frac{5!}{2!}$

Advice (for your future work):

Use scientific notation to get a sense of numbers

(challenging to get a sense of scale in P(50, 40), see previous slide)

Counting "moves":

- count by complement
- count by partition
- count by simplify

Count-by-complement

How many ways are there to order 5 people such that person A is not last?

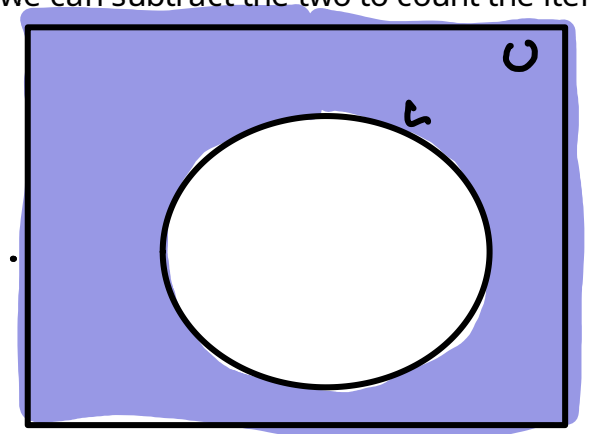
ORDERINGS OF 5 PEOPLE — ORDERINGS OF 5 WHERE A IS LAST = ORDERINGS OF 5 WHERE A IS NOT LAST

$5! - 4! = 5! - 4!$

Count-by-complement

How many ways are there to order 5 people such that person A is not last?

General approach: If we can count everything (U) and all items we're not interested in (L) then we can subtract the two to count the items of interest.



$$|U - L| = |U| - |L|$$

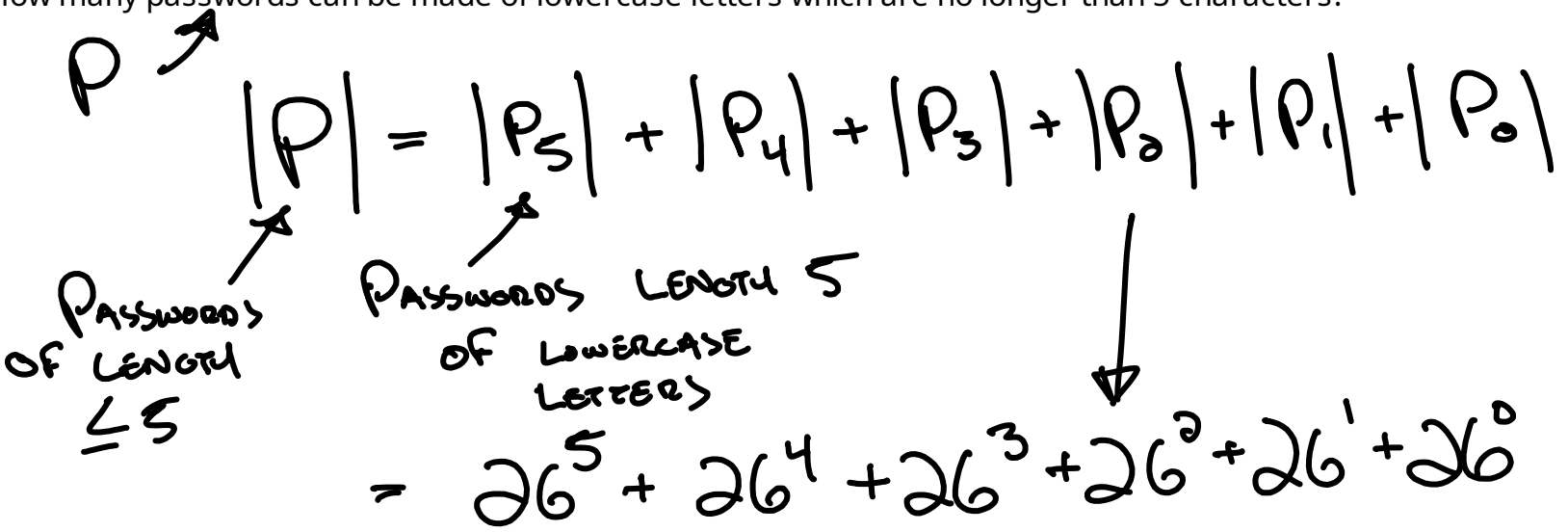
⚠ $|A - B| = |A| - |B|$ NOT TRUE IN GENERAL

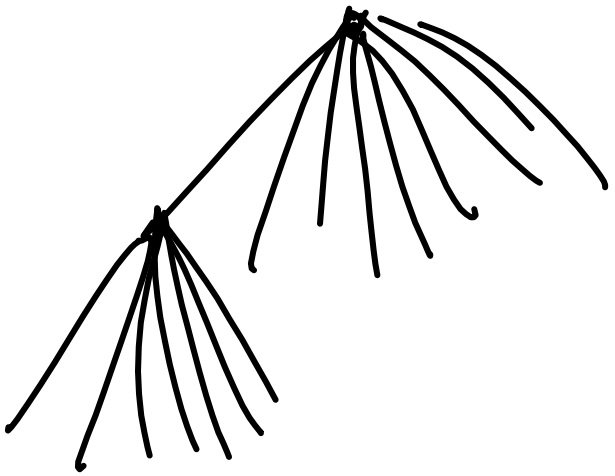
Counting "moves":

- count by complement ✓
- count by partition
- count by simplify

Count-by-partition: motivating example

How many passwords can be made of lowercase letters which are no longer than 5 characters?





26^2 PASSWORDS OF LENGTH 2

$26 \cdot 26$

Partition: definition

Intuition: partition of set A divides its items into subsets where each item in A is in exactly one subset A_i

$A = \{1, 2, 3, 4\}$

$A_1 = \{2, 3\}, A_2 = \{1, 3\}, A_3 = \{4\}$

A IS PARTITIONED BY $A_1 A_2 A_3$

Definition: partition of set A is a set of subsets A_i where every item in A is in exactly one subset

every item in A is in, at most, one subset A_i

subsets are pairwise-disjoint:
intersection of every pair of subsets is empty

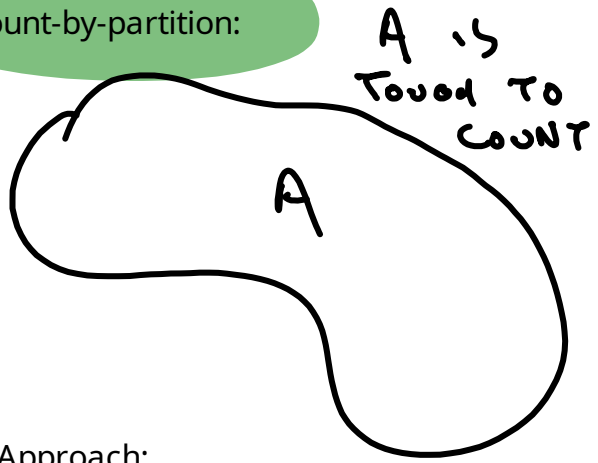
$\forall i, j \quad i \neq j \rightarrow A_i \cap A_j = \emptyset$

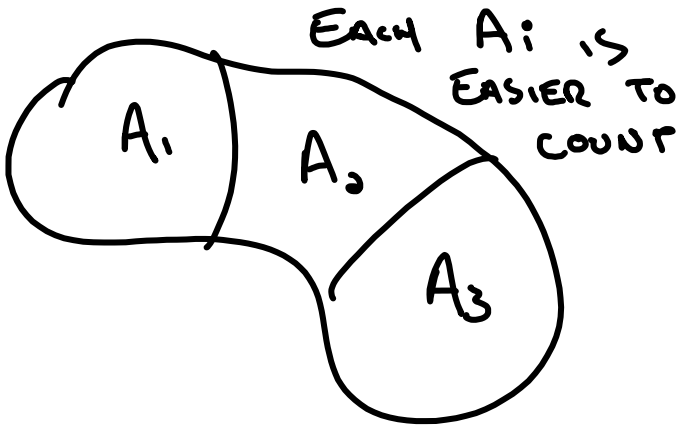
every item in A is in, at least, one subset A_i

union of all subsets A_i is the set A

$A_1 \cup A_2 \cup A_3 \cup \ldots = A$

Count-by-partition:





Approach:
- partition desired set into subsets
- count each subset, add their cardinalities (number of items) together

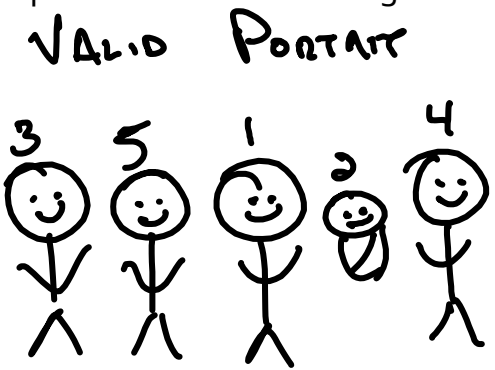
Common Errors: subsets don't partition the set A
- not counting an item of A (violates partition definition: every item of A in at least one subset)
- double counting an item of A (violates partition definition: every item of A in at most one subset)

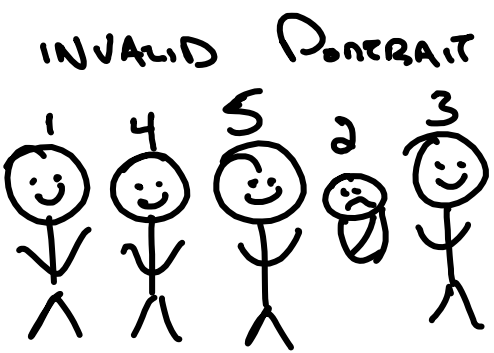
Counting "moves":

- count by complement ✓
- count by partition ✓
- count by simplify

Count-by-simplification:

How many ways can we order 5 family members for a portrait if person 2 is a baby and must be on person 1's immediate right?





$4!_0$

Counting "moves":

- count by complement ✓
- count by partition ✓
- count by simplify ✓

Mea Culpa:

"Count-by-simplification" isn't really a particular approach like others ...

point is, be on the lookout for equivilent problems more easily counted

In Class Activity

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
Assume:

- there are 10 Montague's (Romeo's family, excluding him) who could be in the photo
- there are 7 Capulets (Juliet's family, excluding her) who could be in the photo
- Romeo and Juliet are too busy dancing to be in any picture
- Montagues and Capulets won't get in the same photo (that whole Tybalt / Mercutio thing...)

(hint: partition, simplify a bit)

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

passwords of length 10 - passwords of length 10 start w/ qwerty =

$26^{10} - 26^4$

A W E R T Y _ _ _ _

$25 \cdot 26^9$ → 1ST CHAR NOT Q

= 2ND CHAR NOT W

$6 \cdot 25 \cdot 26^9$

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(hint: partition, simplify a bit)

$P_{E\Omega M}$

$M_0\ M_1\ M_2 \dots M_9$

$C_0\ C_1 \dots C_6$ 10^3

total pictures of either family = pictures from montague only + pictures from capulates only

$P(10,3) + P(7,3)$

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

orderings where person 1 is in the picture + ordering where person 1 is not in the picture

$5 \cdot P(6,4) - 4 \cdot P(5,3) + P(6,5)$

$4 \cdot P(5,3)$

orderings where person 1 is in picture = person 1 in picture (unrestricted) - person 1 in picture (immediately to right of person 2)

$5 \cdot P(6,4)$