CS1800 Day 6

Admin:
- HW 2 due Friday (logic)
- HW 3 released Friday (sets)

Content:
- Sets (subsets, empty set, powerset)
- Set Builder Notation
- Set Operations (Union, Intersection, Complement, Difference)

Sets

A set is a collection of unique objects $\{a, b, c\}$ $= \{a, b, c\}$

MY CURLY BRACES ARE NOT GREAT... SORRY!

$\{1, 2, 3, 4\} = \{1, 2, 3, 4, 4\}$ POOR FORM

$= \{1, 3, 4, 2\}$

AN ITEM IS IN SET OR NOT, NO ITEM IS IN SET MORE THAN ONCE

Example number sets you should be aware of:

Empty set

$\emptyset = \{\ \}$

SET W/ NO ITEMS

Integers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}$

Natural Numbers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

SOMETIMES NOT INCLUDED

Real Numbers

$\mathbb{R}$ CONTAINS

$-2, 0, 1\frac{1}{2}, \pi, e$

Set Builder Notation: one way to express a set

$x \in \mathbb{N}$ $\quad x \notin \mathbb{N}$

$$A = \{ x \in \mathbb{N} \mid (3 \le x) \wedge (x \le 5) \}$$

A is the set of x in natural numbers such that <some predicate>

$0, 1, 2, 3, 4, 5, 6, 7, \ldots$

$A = \{3, 4, 5\}$

In Class Activity: Set Builder Practice

Express the set A by explicitly listing all items it contains

$\ldots -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

$6, 7, 8, \ldots$

$A = \{ x \in \mathbb{Z} \mid |x| < 5 \} = \{ -4, -3, -2, \ldots, 2, 3, 4 \}$

Express the set B using set builder notation

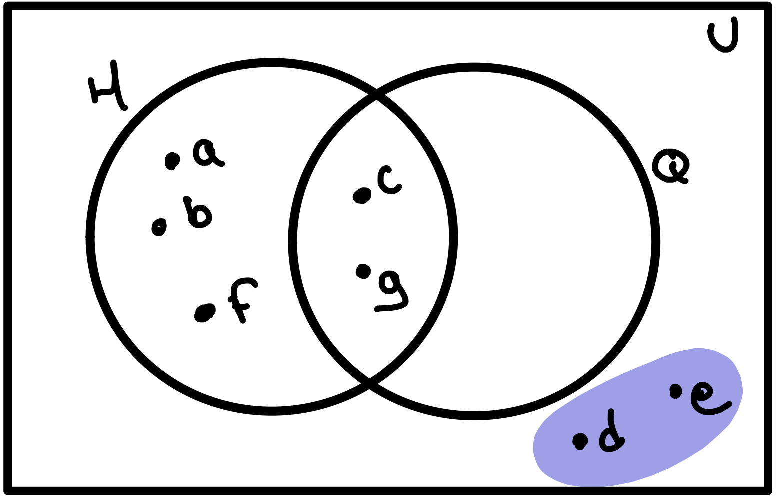
B = set of all natural numbers x which have x mod 3 = 0 and x mod 7 =0 and x < 40

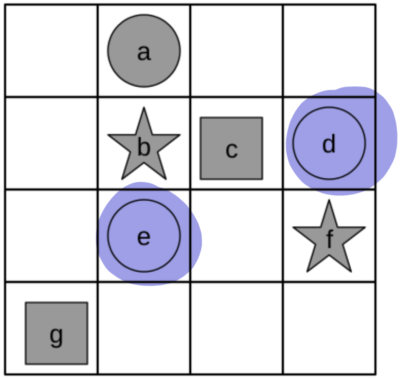
(++ list all of its items)

$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \text{ AND } (x \bmod 7 = 0) \text{ AND } (x < 40) \}$

Venn Diagram: a way of visually representing set membership

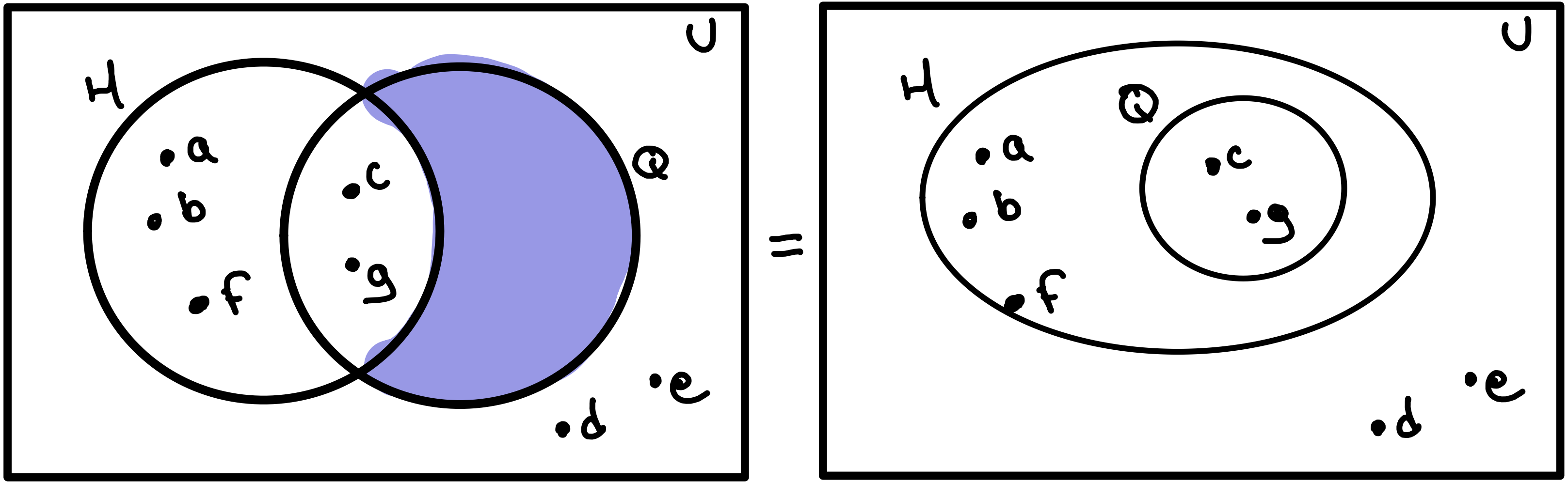
H = set of all sHaded shapes
Q = set of all sQuares
U = Universal set, contains all shapes





Venn Diagram Gotcha: Just because an area exists, doesn't mean it contains any items (may be empty)

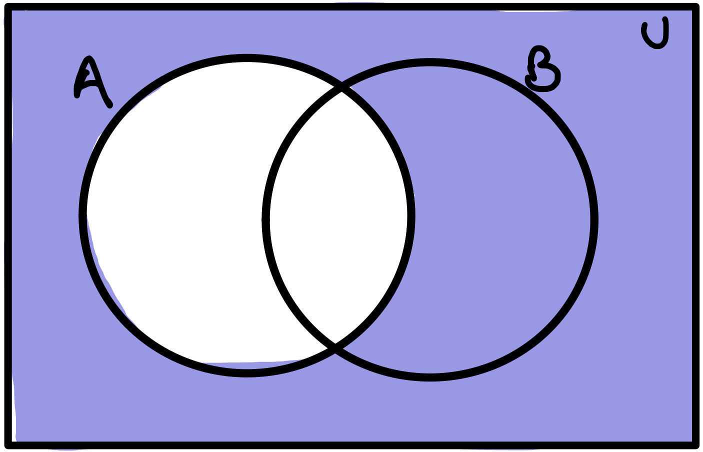
(these Venn Diagrams represent shapes from previous slide)



Generalizable representation:
This classic venn-diagram has a space for any item's set membership

This representation is valid in the special case where one set is contained in another (i.e. Q has no items not in H)

Set Operation: Complement (all the items NOT in some set)



TWO NOTATIONS FOR SAME THING

"NOT IN"

$$\overline{A} = A^c = \{ x \in U \mid x \notin A \}$$

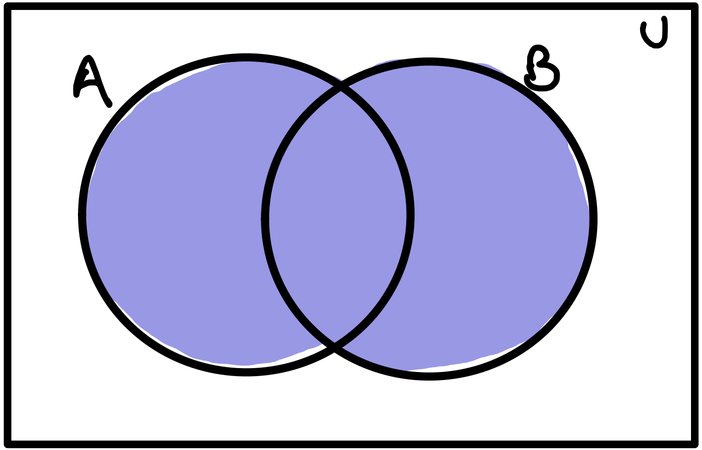
ALL x IN UNIVERSE

SUCH THAT

X IS NOT IN A

Set Operation: Union

(all the items in one set OR another)



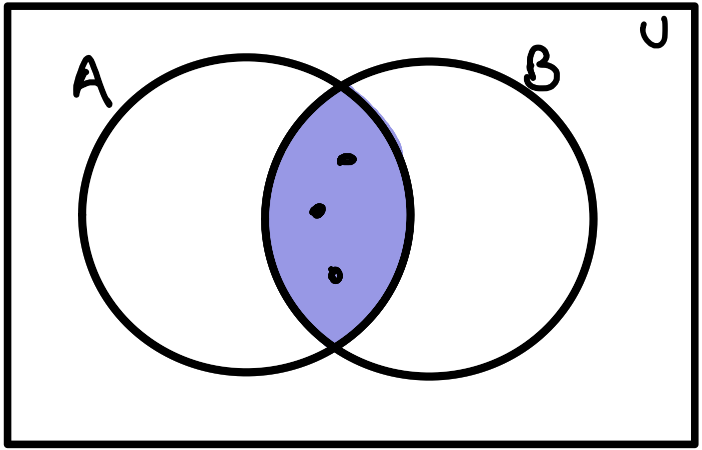
$$A \cup B = \{x \in U \mid x \in A \lor x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

x IS IN A OR x IS IN B

Set Operation: Intersection

(all the items in one set AND another)



$A \cap B = \{x \in U \mid \underline{x \in A} \; \underline{\wedge} \; \underline{x \in B}\}$

ALL x IN UNIVERSE SUCH THAT

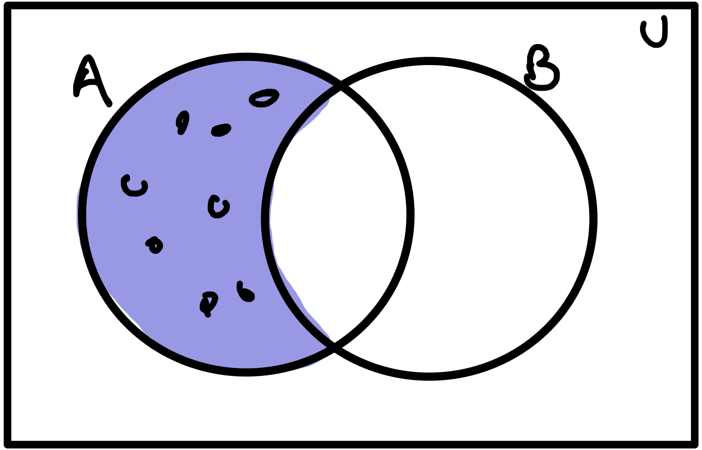
x IS IN A AND x IS IN B

TIP

$\cup$ UNION

$\cap$ INTERSECTION

Set Operation: Difference (All items in one set but not another)

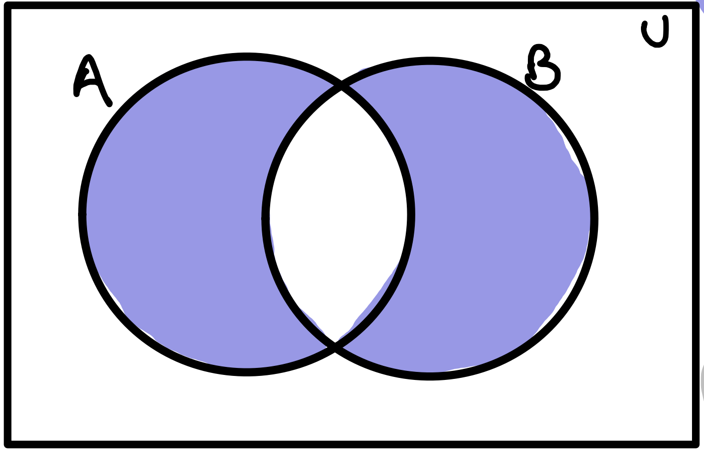


$$A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN A AND X IS NOT IN B

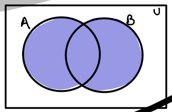
Set Operation: Symmetric Difference (All items in one set XOR another)
(All items in one set or the other, but not both)

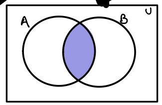


$$A \Delta B = \{x \in U \mid x \in (A \cup B) \wedge x \notin (A \cap B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN $A \cup B$ AND X NOT IN $A \cap B$



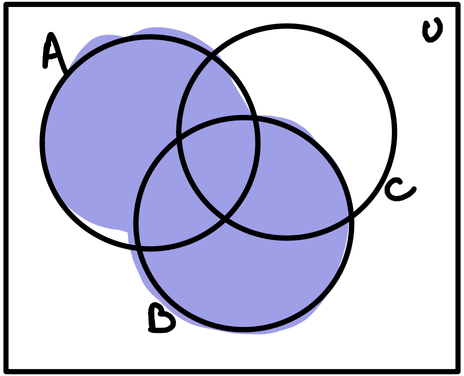


In Class Activity

Shade the indicated areas in each venn diagram

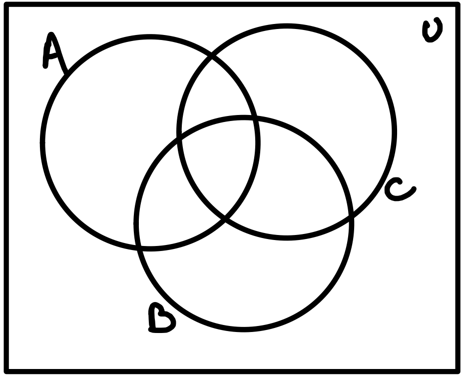
COMPLEMENT OPERATION (NOT SET C)

$(A \cup B) - C$

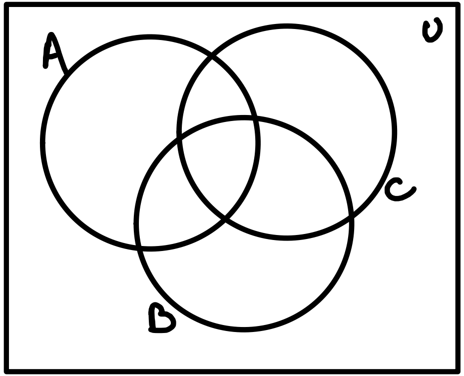


the shaded blue area corresponds to the blue highlighted expression above

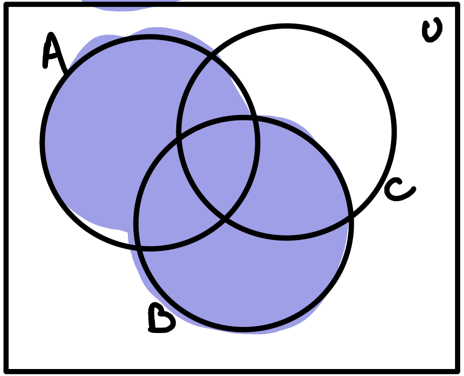
$(A \cap C) \cup B^{c}$



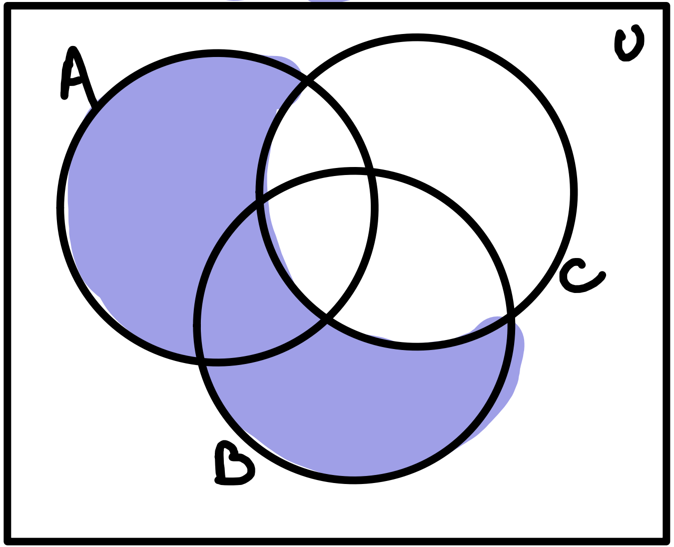
$A \triangle (B \cap C)$



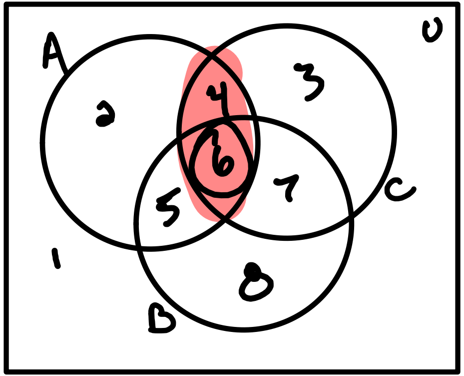
$(A \cup B)$



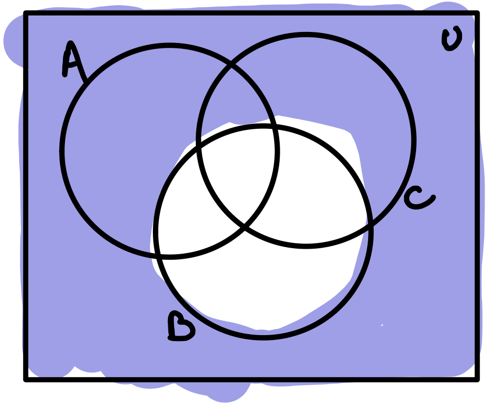
$(A \cup B) - C$



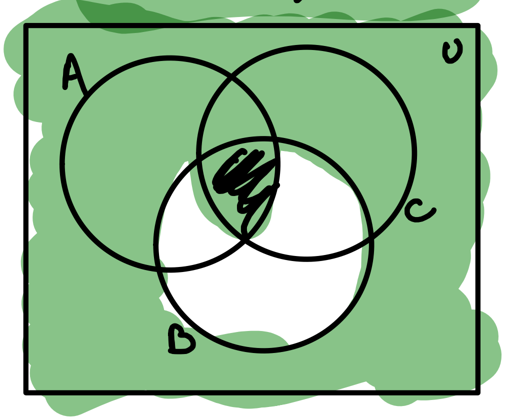
$A \cap C$



B^c

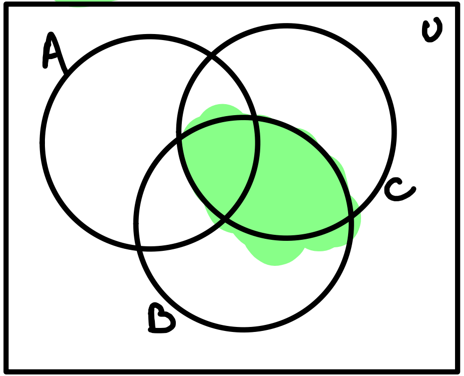


$(A \cap C) \cup B^c$

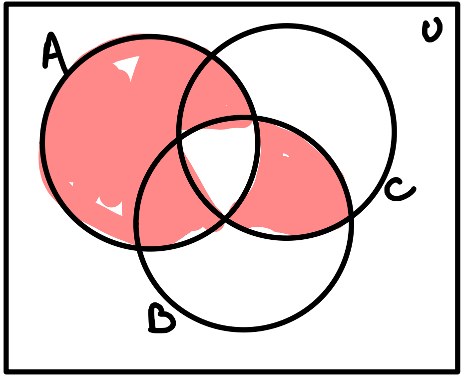


$B \cup B^c = U$

$B \cap C$

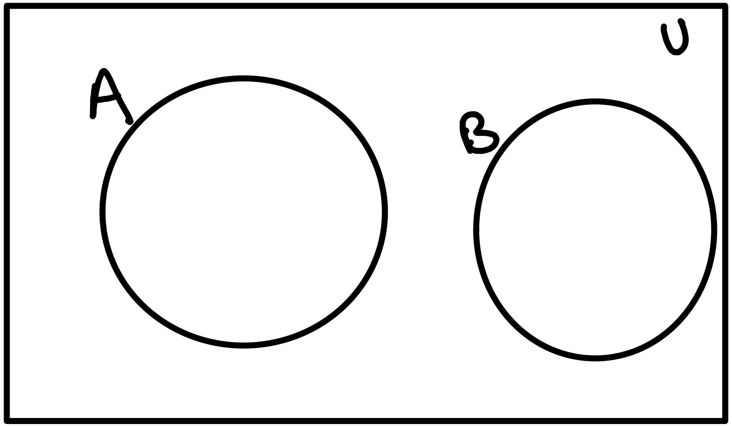


$A \triangle (B \cap C)$



Set Terminology: Disjoint Sets (two sets are disjoint if no item is in both sets)

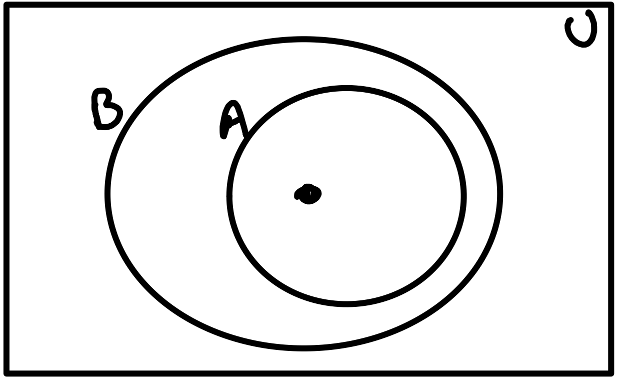
WE SAY A, B ARE DISJOINT IF $A \cap B = \emptyset$



NO ITEM CAN BE IN BOTH A AND B

Set Terminology: subsets

A is subset of B = all items in A are in B



$A \subseteq B = \underline{x \in A} \underline{\to} \underline{x \in B}$

IF X IS IN A THEN X IS IN B

WE ILLUSTRATE LIKE THIS TO SHOW $\underline{A}-\underline{B} = \emptyset$

(THERE IS NO ITEM IN A NOT IN B)

QUIRK: EMPTY SET IS A SUBSET OF ANY SET A

$\emptyset \subseteq A$ FOR ALL SETS A

Set Terminology: Set Equality

$x \in A \leftrightarrow x \in B$

Given sets A, B: we say that A=B if A is a subset of B and B is a subset of A.

$A \subseteq B$

$x \in A \rightarrow x \in B$

ALL X IN A ALSO IN B

$B \subseteq A$

$x \in B \rightarrow x \in A$

ALL X IN B ALSO IN A

INTUITION: A, B HAVE SAME ITEMS

awkward at first look ... but allows for clear set equality proof approach. to show sets A = B:

- show that all items in A are in B and
- show that all items in B are in A

ALSO KIND OF ODD:

$A \subseteq B$ IS TRUE WHEN A, B ARE EQUAL

WHAT LANGUAGE CLARIFIES THAT

$A \subseteq B$ AND B IS "BIGGER" ?

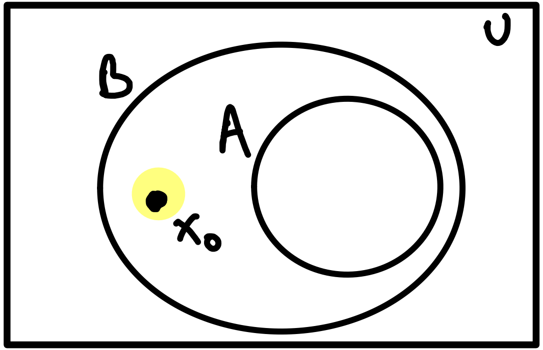
Set Terminology: Proper Subset (one set is contained in another, larger, set)

$A \subset B$ = ALL ITEMS OF A ARE IN B = $A \subseteq B$

AND AND

B CONTAINS SOME ITEM NOT IN A $B - A \neq \emptyset$

"A IS PROPER SUBSET OF B"



NOTICING NOTATION

$$A \subseteq B$$

"SET A IS A SUBSET OF B"

$$x \leq 123$$

UNDERLINE

"MIGHT BE EQUAL"

$$A \subset B$$

"SET A IS A PROPER SUBSET OF B"

$$x < 123$$

Set Terminology: Cardinality (the number of items in a set)

$$A = \{a, b, c, d\}$$

$$|A| = 4$$

Set Terminology: Power Set

The power set of set A is the set of all sets which can be made from items in A

$A = \{1, 2\}$

$P(A) = \{\{1,2\}, \{1\}, \{2\}, \emptyset\}$

$P(A) = \{ \underline{\{1\}}, \underline{\{2\}}, \underline{\{1,2\}}, \underline{\emptyset} \}$

EMPTY SET

In Class Activity (IF TIME)

A = {3, 4, 5}
B = {4, 5}
C = {5}

$D = \{7, \text{'MATT'}, \emptyset, ☺\}$

Compute each of the following

$|A| = 3$
$|A \cup B| = 3$
$|P(C)|$
$|P(B)|$
$|P(A)|$

POWERSET OF A

$C = \{5\}$

$P(C) = \{\{5\}, \emptyset\}$

$|P(C)| = 2$

$B = \{4, 5\}$

$P(B) = \{\{4\}, \{5\}, \{4,5\}, \emptyset\}$

$|P(B)| = 4$

$A = \{3, 4, 5\}$

$P(A) = \{\{3,4,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{3\}, \{4\}, \{5\}, \emptyset\}$

$|P(C)| = 8$

3	4	5	
F	F	F	$\emptyset$
F	F	T	$\{5\}$
F	T	F	$\{4\}$
F	T	T	$\{4,5\}$
T	F	F	$\{3\}$
T	F	T	$\{3,5\}$
T	T	F	$\{3,4\}$
T	T	T	$\{3,4,5\}$

5 is included in subset