CS 1800 Day 4

Admin:
- hw1 due Friday
- hw2 released Friday
- please please read the HW instructions (group members, tagging pages etc)
  (warning: announcement coming reminding everyone midweek too ... apologies for spamming you)

Content:
- logic statements & predicates
- truth tables
- logic operators (AND, NOT, OR)

(just an intro to these topics, we'll do more next lesson too)
- existential / universal quantifier
- conditionals

When should machine:

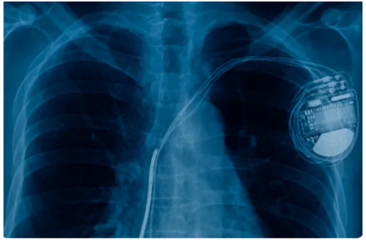
- give a soda
- return change

When should sunroof:

- open
- close

When should pacemaker:

- send pulse to muscle to pump blood?
- shock to restart heart



Logic gives us an unambiguous language to describe behavior

(spoken languages, like english, can be ambiguous)

STATEMENTS

Statement - a sentence which is either true or false

Which of the following are statements?

1. Today is Sept 17

2. "This big wooden horse definitely doesn't have greek soldiers inside"
   - Greeks who just put soldiers in that horse

3. What is your favorite color?

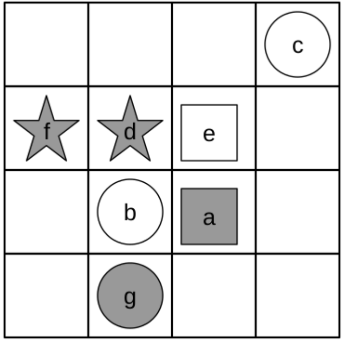
4. There is intelligent life on mars

PREDICATES

Predicate - a statement about one or more variables (i.e. mad libs)

CIRCLE(x)

Tarski World:



circle(x) = True if shape x is a circle, False otherwise

CIRCLE(e) = FALSE

CIRCLE(c) = TRUE

next_to(x, y) = True if shapes x, y are next to each other (diagonals count too), False otherwise

NEXT_TO(b,a) = TRUE

NEXT_TO(f,g) = FALSE

0, 1 True, False

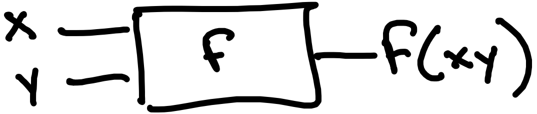
CONVENTION: BITS AND BOOLEANS

0 = FALSE 1 = TRUE

preference: use T and F when talking about Booleans: True and False

we'll use 0 and 1 when discussing circuits (e.g. electronics)

Truth Tables



We'll often describe a function of one or more inputs (e.g. vending machine operation)

A Truth Table specifies an output associated with every possible combinations of inputs

X	Y	f(xy)	
F	F		F
F	T		T
T	F		F
T	T		F

X	Y	Z	g(xyz)
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

A helpful convention:

Order the rows of the truth table as if you're counting in binary

- False becomes 0
- True becomes 1

X	Y
F	F
F	T
T	F
T	T

X	Y	BINARY VALUE
0	0	0
0	1	1
1	0	2
1	1	3

not necesary for credit in this class, but still nice because:

- systematic way to avoid skipping row by accident
- consistent standard allows for easy comparison between us all

Logical Operator: Not (Negation)

Changes truth value

"Not x"

x	$\neg x$
F	T
T	F

Ex

x = "It's raining"

$\neg x$ = "It's not raining"

LOGICAL OPERATOR: AND (CONJUNCTIVE)

ONLY TRUE WHEN **ALL** INPUTS ARE TRUE

"X AND Y"

X	Y	$X \land Y$
F	F	F
F	T	F
T	F	F
T	T	T

D = DRIVER LICENSE PRESENTED

P = PASSPORT PRESENTED

$D \land P$ = DRIVER'S LICENSE AND PASSPORT PRESENTED

LOGICAL OPERATOR: OR (DISJUNCTIVE OPERATOR)

ONLY TRUE WHEN **ANY** INPUT IS TRUE

"X OR Y"

X	Y	$X \lor Y$
F	F	F
F	T	T
T	F	T
T	T	T

D = DRIVER LICENSE PRESENTED

P = PASSPORT PRESENTED

$D \lor P$ = DRIVER'S LICENSE OR PASSPORT PRESENTED

EXCLUSIVE OR: XOR

ONLY TRUE WHEN **EXACTLY ONE** INPUT IS TRUE

"WILL YOU HAVE GREENS OR SOUP?"

G = YOU HAVE GREENS

S = YOU HAVE SOUP

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

NOTICE DIFFERENCE FROM OR

$G \oplus S$ = "EITHER SOUP OR GREENS, NOT BOTH"

INCLUSIVE OR

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

EXCLUSIVE OR

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

"Convention": Most of the time when folk speak "or" they intend the inclusive or

but not all the time ... good luck! ;)

CONVENTION

$$\neg A \vee B = (\neg A) \vee B$$

Assume the negation operation applies to statement immediately to its right.

If the negation applies to multiple statements, use parenthases as below:

$$\neg(A \vee B)$$

Truth tables allow us to build complex expressions in bite-size steps.

GOAL: TRUTH TABLE FOR $(X \vee Y) \wedge \neg Z$

X	Y	Z	$X \vee Y$	$\neg Z$	$(X \vee Y) \wedge \neg Z$
F	F	F	F	T	F
F	F	T	F	F	F
F	T	F	T	T	T
F	T	T	T	F	F
T	F	F	T	T	T
T	F	T	T	F	F
T	T	F	T	T	T
T	T	T	T	F	F

In Class Assignment:

Build a truth table for each of the two expressions below. Results for both might feel familiar, thats ok :)

$\neg(A \vee B)$

A	B	$A \vee B$	$\neg(A \vee B)$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

$\neg A \wedge \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

Logical (Boolean) Equivilence

Two statements are logically equivalent if their truth table columns are identical.

Statements which are logically equivalent:

- always have the same truth value (True or False)
- may be substituted for each other
    - like one does in our familiar algebra (e.g. x = 3 into 10 = x + y)

Example: logically equivalent statements:
"This shape has exactly four sides of equal length at right angles to each other"
"This shape is a square"

Previous slide demonstrates logical equivilence of:

$$\neg(A \vee B) = \neg A \wedge \neg B$$ (De Morgan's Law)

There are other laws too:

- helpful to simplify an expression

- we'll study these alongside
set algebra & circuits, which are
related topics, more to come later …

Associative Laws

$(P \vee Q) \vee R = P \vee (Q \vee R)$

$(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$

Double Negation

$\neg \neg P = P$

DeMorgan's Laws

$\neg(P \vee Q) = \neg P \wedge \neg Q$

$\neg(P \wedge Q) = \neg P \vee \neg Q$

Distributive Laws

$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

Absorption Laws

$P \wedge (P \vee Q) = P$

$P \vee (P \wedge Q) = P$

Complement Laws

$P \vee \neg P = T$

$P \wedge \neg P = F$

Idempotent Laws

$P \vee P = P$

$P \wedge P = P$

Identity

$\text{False} \vee P = P$

$\text{True} \wedge P = P$

Domination:

$\text{True} \vee P = \text{True}$

$\text{False} \wedge P = \text{False}$

Conditional Statement: (AKA Implication)

"IF x THEN y"

Join X	Fun Y	$X \to Y$
F	F	T
F	T	T
T	F	F
T	T	T

x = you join tutoring group
y = you have fun doing math

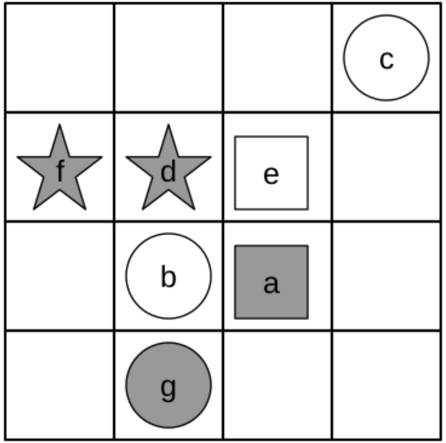
x → y = if you join a tutoring group, then you'll have fun doing math

you haven't joined a tutoring group, statement true by convention (will motivate later)

counter-example: you joined a tutoring group but didn't have fun doing math, statement is False

you joined a tutoring group and had fun doing math

LOGICAL QUANTIFIER: UNIVERSAL (AKA FOR ALL)



∀ x shade(x)

For every object x, x is shaded

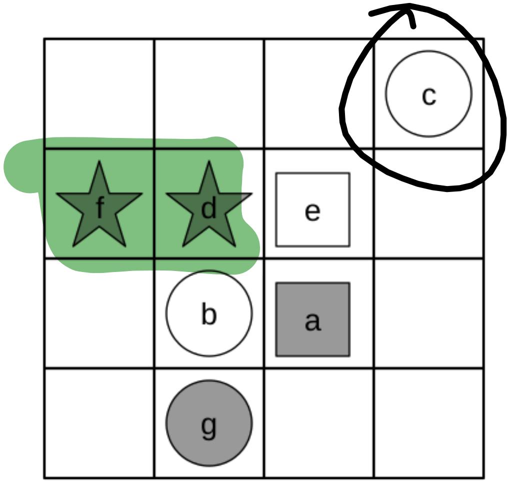
This statement is False for Tarksi world at left,
consider that c is not shaded

Another way of saying the same thing as ∀ x shade(x) is:
shade(a) ∧ shade(b) ∧ shade(c) ∧ shade(d) ∧ ...

in spoken language:
"For any" can be exchanged with: for all, for each, for every, in all cases

Quick Practice

Is following statement true?

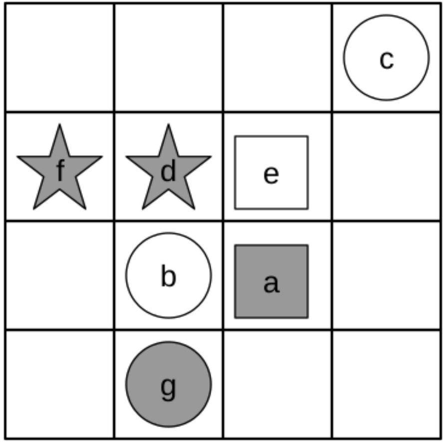


$$\forall x \; \text{Star}(x) \rightarrow \text{Shade}(x)$$

For all shapes x, if x is a star then x is shaded.

$$(\text{Star}(c) \rightarrow \cancel{\text{Shade}(c)})$$
$$\wedge \; (\text{Star}(b) \rightarrow \text{Shade}(b))$$

Logical Quantifier: Existential (AKA "There exists")



$\exists$ x shade(x)

there exists object x where x is shaded

This statement is True for Tarksi world at left, consider that object d is shaded

Another way of saying the same thing as $\exists$ x shade(x) is:

shade(a) ∨ shade(b) ∨ shade(c) ∨ shade(d) ∨ ...

in spoken language:

"there exists" can be replaced with: there is, at least one, it is possible to find, some

USEFUL TIP

$\forall$

FOR ALL

UPSIDE DOWN A

$\exists$

THERE EXISTS

BACKWARDS E

In Class Activity:

hint: be sure to define your predicates & variables

Using logical operators (AND, OR, NOT) quantifiers (for all, there exists) and conditionals (if-then), translate each statement below:

LET X BE STUDENT

SHOES(X) = STUDENT WEARING SHOES

DANCE(X) = STUDENT IS A GREAT DANCER

Logic to english:

$$\exists x \; SHOES(x) \vee DANCE(x)$$

$$\forall x \; DANCE(x) \rightarrow \neg SHOES(x)$$

English to logic (define your own statements & predicates as needed)

- You shall not pass! - Gandalf $\neg PASS(MONSTER)$
- I've got a wallet, keys and a phone in my pocket.
- I never leave the house without my blue shoes or a hat
- "There's no place like home" - Dorothy in Wizard of Oz
- "Everybody loves you when you're 6 feet underground" -John Lennon

$$\forall x \; \neg PASS(x)$$

$\exists x \; SHOES(x) \lor DANCE(x)$

LET x BE STUDENT

SHOES(x) = STUDENT WEARING SHOES

there exists a student x who is either wearing shoes or a good dancer

there exists a student who is either wearing shoes or is a good dancer

DANCE(x) = STUDENT IS A GREAT DANCER

$\forall x \; DANCE(x) \rightarrow \neg SHOES(x)$

for all students, if they are good dancers then they are not wearing shoes.

$w =$

I've got a wallet, keys and a phone in my pocket.

w=I've got a wallet in my pocket, p = i've got a phone in my pocket, k = I've got keys ...

$w \wedge p \wedge k$

- I never leave the house without my blue shoes or a hat

X = MOMENT I LEAVE HOUSE

$\forall x \; SHOE(x) \vee HAT(x)$

$SHOE(x)$ = HAVE B SHOES WHEN I LEAVE HOUSE

$HAT(x)$ = HAVE HAT WHEN LEAVE

- "There's no place like home" - Dorothy in Wizard of Oz

X = A PLACE

$\forall x \neg HOME(x)$

$HOME(x)$ = PLACE X LIKE HOME

- "Everybody loves you when you're 6 feet underground" -John Lennon

$\neg \exists x \, HOME(x)$

X, Y PEOPLE

X PERSON JC TALKING TO

Y PERSON

$\forall x \; \text{DEAD}(x) \rightarrow \forall y \; \text{LOVE}(y, x)$

love(a,b) = true when person a loves person b

$\forall x, y \; x \neq y \; \text{DEAD}(x) \rightarrow \text{LOVE}(y, x)$